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A Note on a Gamma Distribution Computer Program and Computer Produced Graphs

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FIGURES

Symbols used in this report

a	(1) gamma distribution variable dependent on second and third moments
	(2) constant
b	(1) plotting parameter equal to c
	(2) constant
c	(1) plotting parameter with a default option to 0.44
	(2) constant
d	Constant
e	Exponential; 2.7183
f	Function
f_i	Specified constants
g	(1) function
	(2) as subscript, with respect to gamma distribution
h	Geometrical constant
i	(1) sample number
	(2) subscript
j	(1) sample number
	(2) subscript
k	Subscript, such as i or j
ln	Natural logarithm
m	(1) number
	(2) number of iterations
n	(1) number of data
	(2) number of iterations, terms
p	(1) probability of nonzero amounts; NX/NNX
	(2) probability level
q	(1) (1 - p), probability of zero amounts (NNX-NX)/NNX
	(2) constant
r	Variable, reliability index
s	Variable
t	Variable
$\bar{t}$	Average t; the overbar indicates an averaging process
dt	Derivative of t

v	Variable
dv	Derivative of v
x	Variable, here generally $y - \alpha$
dx	Derivative of x
x'	Transformed x, as $(y - \alpha)/\beta$
$\bar{x}$	Average x, nonzero amounts only
y	Variable of function
$\bar{y}$	Average y, nonzero amounts only
z	Variable of integration
A	Numbers (Bernoulli)
D	Determinant
F	Function
G(x)	Gamma distribution function for a measured set excluding zeros - standard cumulative distribution function (cdf)
G^{-1}	Inverse
H(x)	Gamma distribution function for a measured set including zeros
I	Sample number
J	(1) number of duration periods (2) number of data combined
K	Kolmogorov (1933)
K-S	Kolmogorov-Smirnov
M	Moment; subscripts indicate type of moment
ML	Maximum likelihood
NX	Number of data excluding zeros
NNX	Number of data including zeros
P	Probability
P_{n1}	Probability with respect to <u>one</u> tail normal distribution
P_{n2}	Probability with respect to <u>two</u> tail normal distribution
Q	Constant
R	Reliability function
S	Smirnov (1936, 1948)
X	Untransformed variable (i.e., an original datum)
Y	Untransformed variable (i.e., an original datum)

α	Alpha; (1) origin
	(2) probability level for rejection
β	Beta; scale parameter
γ	Gamma; shape parameter
$\hat{\beta}$	Beta hat; maximum likelihood estimate of sample scale parameter
$\hat{\gamma}$	Gamma hat; maximum likelihood estimate of sample shape parameter
$\overset{*}{\beta}$	Beta star; Thom's (1958, 1968) estimate of scale parameter
$\overset{*}{\gamma}$	Gamma star; Thom's (1958, 1968) estimate of shape parameter
ε	Epsilon; error
ζ	Zeta; median, variable
μ	Mu; mean
ρ	Rho; reliable life
σ_s	Sigma; standard deviation
$\sigma^2{}_s$	Sigma square; variance
τ	Tau; quantile
$d\tau$	Derivative of τ
ϕ	Phi; function, standard normal distribution
χ	Chi
χ^2	Chi-square
Γ	Gamma; gamma function
Δ	Delta; increment
Σ	Sigma; summation
$=$	Equal to
$\equiv$	Identically equal to
$\int$	Sigma; Integral
$>$	Greater than
$>>$	Much greater than
$\geq$	Equal to or greater than
$<$	Less than
$\leq$	Equal to or less than
$\cdot$	Multiplication sign
$\overline{}$	Overbar; averaging process
∞	Infinity

A NOTE ON A GAMMA DISTRIBUTION COMPUTER PROGRAM AND COMPUTER PRODUCED GRAPHS

Harold L. Crutcher, Grady F. McKay, and Danny C. Fulbright
National Climatic Center, Environmental Data Service, NOAA
Asheville, N.C.

ABSTRACT. The gamma distribution function may be used as a model for many sets of data. The electronic computer program for this function in the Formula Translator (FORTRAN) IV (1) provides the analytic solution to a set of data, (2) gives the probabilities of exceeding or not exceeding arbitrary amounts, (3) indicates the amounts exceeded or not exceeded for arbitrary probabilities, and (4) provides for computer output to microfilm, drum, or flatbed plotters.

The program, in its general form, permits a maximum of 52 entries which will suffice for those dealing with weekly data through the year. In addition, such as in weekly precipitation studies, the user has the option to compute the two- and three-week duration period distributions in one pass of the data. These computations are done without program change but by appropriate changes in the control cards. This feature is not limited to the study of precipitation data nor to the time intervals used.

An option permits the computation of the required probabilities and inverses when only the scale and shape parameters are given. By appropriate modification, where only one set of data is being examined, the program can accept a string of input data longer than 52.

I. INTRODUCTON

This report presents, for the gamma distribution model,

(1) an improved electronic computer program in the Formula Translator (Fortran) IV to provide an analytic solution for a data set and

(2) computer produced graphs which permit better assessment of the model's applicability.

The five modifications planned for the computer program presented in the previous report, NOAA TR EDS 11 [Crutcher et al. (1973)], were

(1) a subroutine for the determination of an acceptable location (origin) parameter,

(2) a possible subroutine for the debiasing of the maximum likelihood and Thom (1958) shape and scale estimators,

(3) modification of routines to permit calculation of probabilities for shape parameter values, plots, and other x-y type plotters using linear scale plotting,

(4) a subroutine for cathode ray computer output plots, and

(5) a separate program designed for low values of the shape parameter (i.e., $\gamma \leq 1.000$).

The present paper presents the last three modifications as well as the development of an algorithm to extend computing capabilities to shape parameter estimates of about 100. Both the extension of capability to the lower and to the higher values provide the serendipitous dividend of reducing the internal computing time (cpu) more than 50 percent.

NOAA TR EDS 11 provides examples and work sheets for gamma graph paper. One other form of graph paper is illustrated in section XI. In the present paper, output plotter and computer output-to-microfilm routines will provide some graphs with plotted data. These routines may be used to obtain any needed graph paper which may then be duplicated as requested.

The program, in its general form, permits a maximum of 52 entries which will suffice for those dealing with weekly data through the year. In addition, in precipitation studies, the user has the option to compute the two- and three-week duration period distributions in one pass of the data. These computations are done by appropriate changes in the control cards. This feature is not limited to the study of precipitation data.

An option permits the computation of the required probabilities and inverses when only the scale and shape parameters are given. By appropriate modification, where only one set of data is being examined, the program can accept a string of input data longer than 52.

II. THE GAMMA DISTRIBUTION FUNCTION

Many processes produce data distributions that the gamma distribution model describes well. Naturally, considerable literature exists for this distribution. The model serves for reliability life tests and fatigue problems. It offers advantages in the study of many multiple component systems where time-to-failure is an important feature. There are many other applications.

The generalized gamma distribution contains a family of specialized distributions. These cover a wide variety in form and usefulness. Kao (1968) presents a good discussion. As indicated by Kao and others, the gamma distribution may be called the "rope" distribution. A rope does not break until the last fibre has parted. The gamma distribution describing the total of all the fibers of a rope has a shape parameter for the breaking which is the sum of the parameters for the individual fibers or subsets of fibers. In a similar way, other physical processes may be described by the gamma distribution. Haggard, Bilton and Crutcher (1971) discuss such a set which consists of storm processes which produce rain. The specific subset consists of hurricanes which cross the 1,000-ft contour of the Appalachians. Each hurricane consists of entrainment and coalescence of sub-storms.

The gamma distribution is one of the models which is used in reliability techniques when the "rope" philosophy is employed. As presented in NOAA TR EDS 11, the gamma distribution is a form of the Pearson Type III, Pearson (1916). Another form is the Weibull which also can be used in reliability problems. Although not discussed further here, it is useful when the problem is of the "chain" type, analogous to a chain being no stronger than the weakest link.

Pearson (1916) derives the gamma density function (Pearson's Type III) as the solution of a differential equation. The tables edited by Pearson (1922), with subsequent revision through 1957, and those by Pearson and Hartley (1954) permit application of the gamma distribution model to fit and graduate skew data. The above tables permit interpolation for fractional degrees of freedom for the chi-square distribution. Campbell (1923) provides

perhaps the first tabulation of the inverse gamma function if only for integer values. These, of course, are equivalent to the chi-square distribution with integer degrees of freedom equal to twice the gamma values. Salvosa (1930) also provides useful tables. Cohen et al. (1969) extend the tables of Salvosa. Birnbaum and Saunders (1958) derive and use the gamma distribution as one of the models for material life length, which may be likened to the life of a storm or the time-to-failure of precipitation generating processes. Harter (1964, 1969) provides an excellent discussion and extends Pearson's tables. Yet, as Harter says, Pearson's work has no serious contender.

III. THE GENERAL GAMMA DISTRIBUTION FUNCTION

The general gamma distribution with origin parameter $\alpha(-\infty < \alpha < +\infty)$, scale parameter $\beta(\beta > 0)$, and shape parameter $\gamma(\gamma > 0)$ has the probability density function shown in

$$f(y;\alpha,\beta,\gamma) = \beta^{-\gamma}(\Gamma(\gamma))^{-1}(y-\alpha)^{\gamma-1}e^{-(y-\alpha)/\beta};\ y > \alpha;\ -\infty < \alpha, y < +\infty$$

$$\text{and } f(y;\alpha,\beta,\gamma) = 0, \quad y \leq \alpha. \tag{1}$$

The distribution function given by

$$F(y;\alpha,\beta,\gamma) = \int_{\alpha}^{y} f(t;\alpha,\beta,\gamma)dt \tag{2}$$

is for all $y > \alpha$.

Fisher (1922) first develops the maximum likelihood (ML) equation for the solution for the incomplete gamma distribution known commonly as the gamma distribution. It is incomplete in the sense that the integral limits of the function do not range from $-\infty$ to $+\infty$ but from some finite point such as α to $+k$ where k is some real number. If the origin parameter α is zero, this distribution is a special case of the Pearson Type III distribution. The solution of the ML equation as developed by Fisher is difficult. Therefore, Thom (1947) develops approximate solutions. Chapman (1956), Greenwood and Durand (1960), Gupta (1960), and Wilk et al. (1962) provide methods to estimate the gamma distribution parameters. Mooley and Crutcher (1968) discuss the variability of the parameter estimates of two gamma distributions. Schickedanz and Krause (1970) present tests for the scale parameters.

Thom's work leads to fruitful use of the gamma distribution in meteorological, climatological, and hydrological applications. Barger and Thom (1949) furnish an evaluation of drought hazard. Friedman and Janes (1957) provide an estimation of rainfall probabilities. Thom (1958) presents a note on the gamma distribution. Barger et al. (1959) give the chances of receiving selected amounts of n-week precipitation in the north-central region of the United States. The last is the model for a number of subsequent publications. Hartley and Lewish (1959) manage the computer hardware and software for the above study. Thom and Vestal (1968) provide a study of monthly rainfall in the conterminous United States.

IV. PARAMETER ESTIMATION

The gamma distribution (Pearson's Type III) includes the chi-square and the exponential distributions as special cases. Pearson (1922), Thom (1958), and Hahn and Shapiro (1968) discuss this. Most statistical texts briefly discuss this also. The shape parameter γ is equal to one-half the degrees of freedom for the chi-square distribution and is equal to 1 for the exponential distribution, while the scale parameter β is equal to 1 in the standardized case as well as the last two cases.

Barger et al. (1959) provide plotting paper where the arguments are the mean and the variate. Overlaid straight lines represent probabilities. Each value of the shape parameter γ requires a separate graph. In the same paper, Thom's distribution curves, also prepared from Pearson's tables in 1957, appear. The probability and the variate divided by the scale parameter are the arguments with the shape parameter being overlaid in curved lines over the argument plot.

Wilk et al. (1962) provide techniques to estimate the scale and shape parameters, and they indicate that computer routines are available to provide graphical plots in terms of the quantile probabilities of the distribution and scale units. The theoretical line of best fit is then a straight line. These authors provide a brief set of tables that allows the person with a desk calculator, slide rule, or paper and pencil to interpolate required probability values and scale values and to make a plot of the data against the line obtained from the estimate of the scale and shape parameters. Roy et al. (1971) incorporate the above paper.

Thom (1968) presents direct and inverse tables of the gamma distribution. Thom's tables fill in areas not covered by the Wilk et al. (1962) tables and repeat other portions for verification. Crutcher et al. (1973) provide graphical techniques to estimate the scale and shape parameters.

V. ORIGIN

The origin or location parameter α in equation (1) usually is set to zero. However, there are cases where the origin is not zero. Elderton (1953) uses Pearson's moments to locate an origin from which the other parameters of the distribution may be measured. The necessary statements follow:

$$\alpha = \text{origin} = \text{mode} - a,$$

$$a = (2M_2^2/M_3) - (M_3/2M_2),$$

$$\text{mode} = \bar{t} - (M_3/2M_2),$$

and

$$\text{origin} = \bar{t} - ((2M_2^2)/M_3) \qquad (3)$$

where M_2 and M_3 are the second and third moments from the mean of the distribution. Barger (1964) discusses this. The expression $\bar{t} - (2M_2^2/M_3)$ does not ensure a positive location estimate even though the observed values are all positive. In some cases, the estimate may be higher than the lowest observed and recorded value in the data set.

Pitman's (1938) estimator for the location (origin) parameter is a minimum variance unbiased estimator if the scale and shape parameters are known. These parameters usually are not known and must be estimated. Pitman's technique is not examined further in this report.

Hastings (1955) provides an equation for the estimation of the origin. Greenwood and Durand (1960) also study the estimation of the location parameter. Chapman (1956) provides a tabular aid for iterative procedures to solve for the origin parameter in the untruncated case. He indicates an additional procedure for the truncated case, providing there is sufficient supplementary information. These iterative procedures are not examined in this report.

Blischke (1975, 1971, and in prior studies) pursues the solution to the problem. Blischke encountered the same difficulties in the estimation process as is mentioned for the Elderton estimator. This, of course, blocks the calculation of maximum likelihood estimators for the shape

and scale parameters or in any estimation process where logarithms are used. Blischke suggests that the lowest value be used as the origin where the estimated origin is above the lowest observed datum. This is the maximum likelihood estimator for the origin. In NOAA TR EDS 11, Crutcher et al. (1973) found that fit to the gamma distribution may be rejected when this is done, even though a value slightly lower than the datum as the estimator for the origin is used. Blischke (1975) brings the work on estimation of the origin parameter up to date.

The program presents several options for the origin. The default option uses zero as the origin. Such a case would be zero for measured precipitation. If prior experience or theoretical considerations indicate the value(s) of the origin parameter(s), this option is entered in a control card that replaces the default option. A third option uses the lowest value less a small amount to ensure the positive number needed for the logarithms used in the maximum likelihood or Thom's estimators. Additionally, if the lowest value occurs more than once, this value becomes the origin.

The program processes the mixed distribution that consists of two categories, the lower bound and the values above the bound. Categories by non-occurrence, such as zero precipitation, and the distribution of measurable precipitation above the bound is such a mixed distribution.

Regarding the bias in the estimators of maximum likelihood, it is of interest to refer to Blischke's work and to that of Shenton and Bowman (1970), Fisher (1922), and Thom (1957, 1958). Here we reproduce the comments of Shenton and Bowman:

"In this note we show that Thom's statistics are:

a) slightly biased, no matter how large the sample; however this bias is almost negligible for $\gamma >> 0$, and indeed is only of any real importance if γ is small (say less than 0.1 approximately); the bias in finite samples is about the same as for the maximum likelihood estimators;

b) superior to the maximum likelihood estimators because their variances are less in large sample theory; there is evidence that this property holds in finite samples also;

c) about as near to normality (as measured by skewness and kurtosis) as the maximum likelihood estimators; actually the distribution of $\overset{*}{\beta}$ is generally nearer to the normal form than that of $\hat{\beta}$."

In the above, the $\hat{\beta}$ and $\overset{*}{\beta}$ are respectively the maximum likelihood and Thom estimators of scale parameters.

Removal of bias in the estimators is not attempted in this program and report. Woodward and Gray (1975) discuss for the scale parameter the "Minimum Variance Unbiased Estimation in the Gamma Distribution," the "UMVU," through the use of a new infinite series. This technique is not examined in the present paper. Anderson and Ray (1975) present modifications designed to reduce the bias and mean square error for the two-parameter gamma distribution. Such will be examined later. In view of the large variability of the estimates (Andrews and Barger, 1956; Mooley and Crutcher, 1968), removal of the bias may or may not be appropriate.

With the origin α obtained, the following expression is pertinent:

$$x = y - \alpha. \tag{4}$$

Then (1) becomes

$$f(x;0,\beta,\gamma) = \beta^{-\gamma}(\Gamma(\gamma))^{-1}\, x^{\gamma-1}e^{-x/\beta}, \quad 0 < x < \infty$$

$$\text{and } f(x;0,\beta,\gamma) = x \leq 0. \tag{5}$$

Thom (1968 and in his earlier papers) utilizes this form. As shown by Thom (1958) and by Wilk et al. (1962), if the variate x assumes a transform by division of the scale parameter β, the distribution function develops as

$$F(x';0,1,\gamma) = (\Gamma(\gamma))^{-1} \int_0^{x'} \tau^{\gamma-1}\, e^{-\tau}\, d\tau, \quad x' > 0$$

$$\text{and } F(x';0,1,\gamma) = 0, \quad x' \leq 0\ , \tag{6}$$

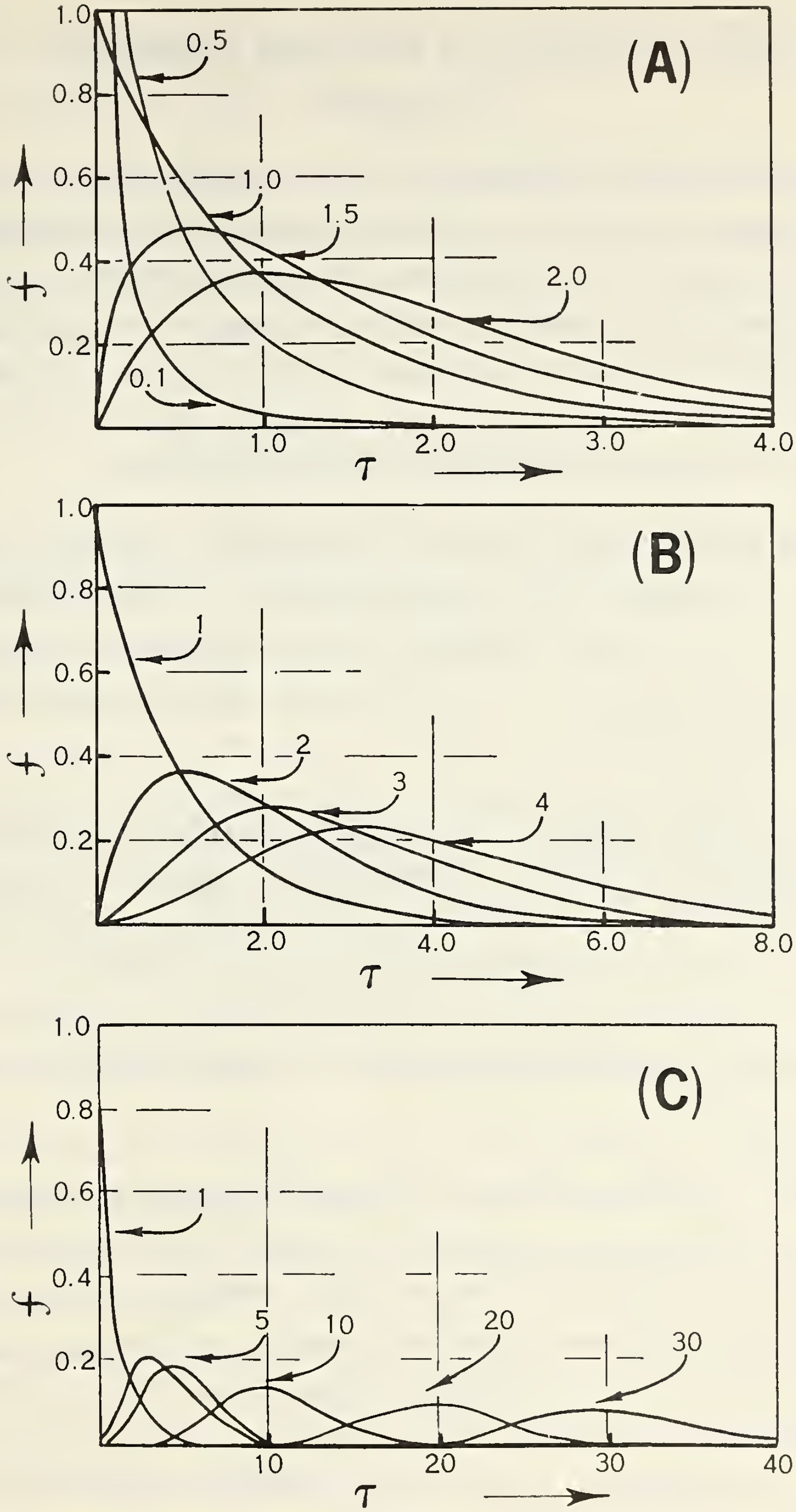
that is a standard form with $\alpha = 0$ and $\beta = 1$ and is positive when $x > 0$.

Figure 1 provides a picture of the effect of the shape parameter and scale parameter on the function curves. Here, the standardized scale (frequency) is plotted against the quantile τ. Curves for shape parameters (A) 0.5, 1.0, 1.5, and 2.0, (B) 1, 2, 3, and 4, and (C) 1, 5, 10, 20, and 30 illustrate the effect. The shape parameter for 1.0 is shown on each subset, but the horizontal scale has been compressed. Hahn and Shapiro (1968) and Falls (1971) provide illustrations for other combinations of the scale and shape parameters. Reference to χ^2 curves also may be made. Where $\gamma = 1$, this is the same as the exponential distribution and the χ^2 with two degrees of freedom. Stated somewhat differently, the random variable $(1/2)\chi^2_{2\gamma}$ with 2γ degrees of freedom has the gamma density function with the scale parameter equal to one and the shape parameter equal to γ.

Wilk et al. (1962) and Thom (1968) present the numerical methods to obtain the estimates of the gamma distribution scale and shape parameters β and γ. Masuyama and Kuroiwa (1951) provide a table for the likelihood solution of the gamma distribution. Those papers provide more detail. As Barger et al. (1959) indicate, the estimates of the parameters are subject to rather large variations due to sampling and estimating errors. Mooley and Crutcher (1968) discuss the variance of the probabilities of exceeding stated amounts based on work of Andrews and Barger (1956).

For a particular gamma variate distribution, the product of the shape and scale parameters equals the mean of the nonzero quantities. That is, $\overset{*}{\beta}\overset{*}{\gamma} = \bar{y}$. If $y_1, y_2, \ldots, y_n$ are independent gamma variates with shape parameters equal to $\gamma_1, \gamma_2, \ldots, \gamma_n$, then $Y = \sum_{i=1}^{n} y_i$ is a gamma variate with a shape parameter equal to $\sum_{i=1}^{n} \gamma_i$ (Kenney and Keeping, 1951 and Lancaster, 1969). This provides a useful tool for combining parameter estimates, thereby reducing the computation that would be required if the original data sets were combined. The division of the mean of the total set by the new shape parameter estimate provides the new scale parameter estimate.

An option is available in the computer program discussed below that permits the calculation of probabilities from the input value of the parameter estimates in lieu of entry of original data with subsequent calculation of the estimates.



$$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1}\, e^{-(y-\alpha)/\beta}$$

FIGURE 1. SELECTED GAMMA DISTRIBUTION FUNCTION CURVES.

VI. PROCEDURES FOR SMALL SHAPE PARAMETERS

1. General

One of the difficulties encountered in the computer program developed for NOAA TR EDS 11 was the increasing failure rate to converge as shape parameters, when less than 1, became less and less. Such failure rate also was noted as the shape parameter, when greater than 40, became larger and larger. Occasional intermittent failure at isolated points at higher and lower probability levels also occurred. This last is not extremely important for sufficient nearby points are available to permit easy interpolation.

This section and the next section, incorporated in the new program developed and presented in this paper, resolve all three difficulties. But more than that, the internal computing time is decreased by more than 50 percent. These developments are extensions of the Williams (1946) and Pinkham (1962) techniques.

As no direct inversion exists for the gamma distribution, the extraction of quantiles presents problems. Approximation techniques have their associated difficulties. In general, as the shape parameter decreases, the determination of quantiles becomes increasingly more difficult. Thom (1968) gives tables of quantiles which were derived using approximation methods. He also recognized the difficulty of quantile determination for small values of the shape parameter.

Pinkham (1962), following Williams (1946), presented an approximation for the evaluation of the gamma probability integral which used the normal distribution. Using the inverse of the Williams-Pinkham techniques, tables of conversion factors have been determined whereby, with few exceptions, quantiles can be computed for 52 selected probabilities for γ = 0.1[0.1]4.4. In addition, these conversion factors are given for γ = 0.01 to satisfy boundary requirements. The number 52 is arbitrary and was chosen to fit a sequence of 52 weekly values for a year. For the region of the tables for which conversion factors yield unreliable results, another approximation has been used for quantile determination. In essence, this section is an extension of the Williams-Pinkham techniques. Conversion factors for shape parameters

other than those given may be determined using an interpolating parabola. These values are given in table 1 of appendix 1.

Tables of quantiles computed for 21 probability levels and probabilities for selected quantiles are presented in tables 2 and 3 of appendix 1. These are given for γ = 0.01[0.01]0.10, 0.10[0.05]1.0, and 1.0[0.1]1.5. These, over parts of the ranges, duplicate those presented by Wilk et al. (1962) and Thom (1968).

2. The gamma distribution

For continuity purposes, equations (1) and (6) are repeated here. The probability density function for the gamma distribution is

$$f(y;\alpha,\beta,\gamma) = \beta^{-\gamma}(\Gamma(\gamma))^{-1}\,(y-\alpha)^{\gamma-1}\,e^{-(y-\alpha)/\beta}\;,\; y > \alpha \geq 0 \tag{1}$$

and $f(y;\alpha,\beta,\gamma) = 0,\; y \leq \alpha$,

where y denotes the random variable, α is the location parameter (generally $\alpha = 0$), β is the scale parameter, and γ is the shape parameter. $\Gamma(\gamma)$ is the gamma function where γ, the shape parameter, is used as the argument. Letting $\alpha = 0$ and defining $x' = (y-\alpha)/\beta$, the distribution function becomes

$$F(x';0,1,\gamma) = (\Gamma(\gamma))^{-1}\int_0^{x'} \tau^{\gamma-1}e^{-\tau}d\tau,\; x' > 0 \tag{6}$$

and $F(x';0,1,\gamma) = 0,\; x' \leq 0$,

where τ is the quantile and x' is a transformed variate.

While the evaluation of (6) may be done using Pearson's expansion (1957), the evaluation may also be done using an approximation given by Pinkham (1962). Following Williams (1946), Pinkham developed an approximation to the gamma distribution probability integral (6) which entails the use of the standard normal distribution. Modifying Pinkham's notation, we can write

$$(\Gamma(\gamma))^{-1}\int_0^{t} \tau^{\gamma-1}e^{-\tau}d\tau \simeq [P(h\sqrt{t})]^{2\gamma}\;, \tag{7}$$

where h is a geometrical constant, $t \equiv x'$,

$$P(h\sqrt{t}) = (\sqrt{2\pi})^{-1}\int_{-h\sqrt{t}}^{+h\sqrt{t}} e^{-z^2/2}\,dz\;, \tag{8}$$

and where z replaces τ as a variable of integration. Pinkham provides tables of h values for selected shape parameters and probabilities.

3. The inverse of the gamma distribution

The use of Newton's Approximation to evaluate the inverse gamma distribution has been discussed by Thom (1968). An examination of (7) suggests that, given h values, quantiles of the gamma distribution may be derived by use of the inverse normal distribution function.

For convenience, the following expressions are redefined as

$$P_g(t) \equiv F(t;\gamma)$$

$$\text{and } P_{n2}(x) \equiv P(h\sqrt{t}) \ .$$

Now consider the approximation (7) to be

$$P_g(t) \approx [P_{n2}(x)]^{2\gamma}$$

and take the $(1/2\gamma)$ root of both sides. This yields

$$[P_g(t)]^{1/2\gamma} \approx P_{n2}(x) \ . \tag{9}$$

Rewriting the right side of (9) in terms of a one-tail normal distribution gives $(P_{n2}(x) + 1)/2$. To maintain the equality expressed in (9), the left side is treated in the same manner. Defining $P_{n1}(x) = (P_{n2}(x) + 1)/2$, expression (9) may be written as

$$([P_g(t)]^{1/2\gamma} + 1)/2 \approx P_{n1}(x) \ , \tag{10}$$

where

$$P_{n1}(x) = (\sqrt{2\pi})^{-1} \int_{-\infty}^{x} e^{-z^2/2}\, dz \ . \tag{11}$$

Now $x \equiv h\sqrt{t}$ can now be computed by inverting (11). Given h values for the specified shape parameter, the quantile t may be determined in a straightforward manner, i.e., $t = (x/h)^2$. Furthermore, if one is willing to accept t as an initial estimate of the quantile, an appropriate approximation technique improves the estimate of the quantile. For quantiles presented in appendix 1, the Newton Approximation substantially improves quantile estimates obtained by inverting (11).

4. Determination of h values

By using Pinkham's approximation in conjunction with quantiles of the gamma distribution (Wilk, Gnanadesikan, and Huyett, 1962), tables of h values have been computed for 52 selected probabilities for $\gamma = 0.1[0.1]4.4$ and for a lower boundary $\gamma = 0.01$. Table 1 presents these h values as an extension to those of Pinkham. These values were determined by "guessing" an h, computing a quantile, and subsequently a probability for the quantile. The "guessed" h value was then incremented by some Δh, e.g., 0.1×10^{-3} and the resulting h value was used to determine a new quantile from which the probability again was computed. This incrementation and subsequent computation of quantiles and probabilities was continued until the relative error in the probability was at a minimum for the increment being used. The initial guess for each shape parameter was based primarily on Pinkham's values.

Values of h for extremely low values of the shape parameter and extremely low probabilities should be used cautiously. In general, h values for those combinations of γ and $P_g(t)$ for which $[P_g(t)]^{1/2\gamma} < 0.5 \times 10^{-10}$ may yield unreliable values of quantiles. Another method is given later which allows more reliable quantile computations.

5. Interpolation of h values

For the determination of h values for shape parameters other than those presented in this paper, e.g., 0.336, an interpolating parabola (Southworth and Deleeuw, 1965) was found to yield h values of sufficient accuracy. The general equation of this parabola is

$$h(\gamma) = a_1\gamma^2 + a_2\gamma + a_3 , \qquad (12)$$

where a_1, a_2, and a_3 are constants to be evaluated. As three constants are to be determined, a system of three equations is required. This system may be obtained by sutstituting three functional values into the polynomial. The solution to any system of linear algebraic equations can be obtained using determinants and applying Cramér's Rule (Michal, 1947 and Perlis, 1952). Since 3rd order determinants are involved here, they may be evaluated by developing them by row or by column. If the order were higher, it would

be more advantageous to employ some other technique of evaluation to solve the system of equations. One such method would be the Gaussian Algorithm, which is a systematic elimination method.

Let us examine the solution of a system of three equations. Let this system be

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= y_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= y_2 \quad , \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= y_3
\end{aligned}
$$

where a_{11}, a_{12},....a_{33} are the coefficients of unknowns x_1, x_2, and x_3 and y_1, y_2, and y_3 denote function values. The determinant of the coefficients is

$$
D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} .
$$

Solutions x_1, x_2, and x_3 will exist provided that $D \neq 0$ and the system is non-homogeneous, i.e., y_1, y_2, and $y_3 \neq 0$.

Three additional determinants are required to use Cramér's Rule with a system of three equations. These are

$$
D_1 = \begin{vmatrix} y_1 & a_{12} & a_{13} \\ y_2 & a_{22} & a_{23} \\ y_3 & a_{32} & a_{33} \end{vmatrix}, \; D_2 = \begin{vmatrix} a_{11} & y_1 & a_{13} \\ a_{21} & y_2 & a_{23} \\ a_{31} & y_3 & a_{33} \end{vmatrix}, \text{ and } D_3 = \begin{vmatrix} a_{11} & a_{12} & y_1 \\ a_{21} & a_{22} & y_2 \\ a_{31} & a_{32} & y_3 \end{vmatrix}.
$$

By Cramér's Rule, the solutions are

$$x_1 = D_1 D^{-1}, \; x_2 = D_2 D^{-1}, \text{ and } x_3 = D_3 D^{-1}.$$

The evaluation of the required determinants can be done easily by expansion

by minors as only 3rd order determinants are involved. Evaluation of D using the first row yields

$$D = a_{11}(a_{22}a_{33} - a_{32}a_{23}) + a_{12}(a_{31}a_{23} - a_{21}a_{33}) + a_{13}(a_{21}a_{32} - a_{31}a_{22}).$$

Expansion of the determinants D_1, D_2, and D_3 gives

$$D_1 = y_1(a_{22}a_{33} - a_{32}a_{23}) + a_{12}(y_3a_{23} - y_2a_{33}) + a_{13}(y_2a_{32} - y_3a_{22})$$

$$D_2 = a_{11}(y_2a_{33} - y_3a_{23}) + y_1(a_{31}a_{23} - a_{21}a_{33}) + a_{13}(a_{21}y_3 - a_{31}y_2) \quad .$$

$$D_3 = a_{11}(a_{22}y_3 - a_{32}y_2) + a_{12}(a_{31}y_2 - a_{21}y_3) + y_1(a_{21}a_{32} - a_{31}a_{22})$$

For an example, consider the determination of an h value for $\gamma = 0.336$ at the 0.001 probability level. From table 1 appropriate values are taken and placed into (12) to give the system of equations,

$$\begin{aligned} 0.09a_1 + 0.3a_2 + a_3 &= 1.5009 \\ 0.16a_1 + 0.4a_2 + a_3 &= 1.4554 \quad . \\ 0.25a_1 + 0.5a_2 + a_3 &= 1.4142 \end{aligned}$$

The determinant of the coefficients is

$$D = \begin{vmatrix} 0.09 & 0.3 & 1 \\ 0.16 & 0.4 & 1 \\ 0.25 & 0.5 & 1 \end{vmatrix} \quad .$$

The remaining determinants are

$$D_1 = \begin{vmatrix} 1.5009 & 0.3 & 1 \\ 1.4554 & 0.4 & 1 \\ 1.4142 & 0.5 & 1 \end{vmatrix} ,$$

$$D_2 = \begin{vmatrix} 0.09 & 1.5009 & 1 \\ 0.16 & 1.4554 & 1 \\ 0.25 & 1.4142 & 1 \end{vmatrix} ,$$

and

$$D_3 = \begin{vmatrix} 0.09 & 0.3 & 1.5009 \\ 0.16 & 0.4 & 1.4554 \\ 0.25 & 0.5 & 1.4142 \end{vmatrix} \quad .$$

Evaluating these determinants and applying Cramer's Rule gives

$$a_1 = 0.214999, \quad a_2 = -0.6055000, \quad \text{and } a_3 = 1.663200.$$

Substituting these values into (12) gives the required interpolating parabola, $h(\gamma) = 0.214999\ \gamma^2 - 0.6055000\ \gamma + 1.663200$. Thus, for $\gamma = 0.336$, $h = 1.4840$.

6. Calculation of the argument $h\sqrt{t}$ and t

Now that the interpolation of h values has been discussed, let us return to the calculation of $h\sqrt{t}$. As previously stated, this value may be obtained by taking the inverse of (11). $P_{n1}(x)$ has a value given by (10). An initial estimate of $h\sqrt{t}$ can be found using Hasting's (1955) rational polynomial approximation,

$$x_0 = r - \frac{c_0 + c_1 r + c_2 r^2}{1 + d_1 r + d_2 r^2 + d_3 r^3} + \varepsilon\ (Q)\ , \tag{13}$$

where $r = [\ln(1/Q_2)]^{\frac{1}{2}}$, $Q = 1 - P_{n1}(x)$, and $|\varepsilon(Q)| < 4.5 \times 10^{-4}$.

The constants are

$$\begin{array}{ll} c_0 = 2.515517 & d_1 = 1.432788 \\ c_1 = 0.802853 & d_2 = 0.189269 \\ c_2 = 0.010328 & d_3 = 0.001308 \end{array} .$$

Using x_0 as a starting point, Newton's Approximation Method may be employed to obtain a better estimate of the true value of x. Thus, write $f(x) = \phi(x) - P_{n1}(x)$ and calculate

$$x_i = x_{i-1} - (f(x_{i-1})/f'(x_{i-1})),\ i = 1,\ldots,m\ . \tag{14}$$

Let $\phi(x)$ denote the standard normal distribution integral,

$$\phi(x) = (\sqrt{2\pi})^{-1} \int_{-\infty}^{x} e^{-z^2/2}\, dz\ . \tag{15}$$

Then $f'(x) = \phi'(x) = (\sqrt{2\pi})^{-1}\ e^{-x^2/2}$. (16)

If (in using Newton's Approximation (14)) $m = 2$, $P_0 = 10^{-s}$, and $P_0 < P_{n1} < 1 - P_0$, then (as Milton and Hotchkiss (1969) point out), the error $\varepsilon \leq 10^{-(11-s)}$ for $1 \leq s \leq 9$. While m may be specified, it is better to halt computations when either $[x_i - x_{i-1}]$ or $[\phi(x_i) - \phi(x_{i-1})]$ is less than some specified value.

The use of Newton's Approximation (14) requires the evaluation of the normal probability integral (15) for each successive approximation. This evaluation may be accomplished by using a power series or a polynomial approximation. Although quantiles presented in this paper were computed using a routine which employed the power series given below, a polynomial has also been used which can provide the same accuracy as the power series and effectively reduce computation time. The power series is

$$\phi(x) = 0.5 + \phi'(x) \sum_{n=0}^{\infty} \frac{x^{2n+1}}{1 \cdot 3 \cdot 5 \cdot \cdot \cdot (2n+1)}, \qquad (17)$$

where $\phi'(x)$ is the normal density function defined by (16).

The following polynomial expression (Hastings, 1955) is used in the program presented in this report for computing normal probabilities in conjunction with Newton's Approximation:

$$\phi(x) = 1 - \phi'(x)\,(f_1\zeta + f_2\zeta^2 + f_3\zeta^3 + f_4\zeta^4 + f_5\zeta^5) + \varepsilon(x) \qquad (18)$$

where $\zeta = (1+qx)^{-1}$ and $|\varepsilon(x)| \leq 7.5 \times 10^{-8}$.

The constants are

$$q = 0.2316419$$

$f_1 = 0.319381530$ $\qquad f_4 = -1.821255978$

$f_2 = -0.356563782$ $\qquad f_5 = 1.330274429$.

$f_3 = 1.781477937$

When x_m is returned from (14), it is the value $h\sqrt{t}$. The value h may be obtained from the appropriate table or by interpolation as described previously. Division of x_m by h and squaring gives the desired quantile t. As an example, compute the quantile for $P_g(t) = 0.001$ and $\gamma = 0.336$. It was determined earlier that for this probability and γ, h was 1.4840. Thus, from (11) $P_{n1}(x) = (1 + ((.001)^{1/2(0.336)}))/2 = 0.5000171666$. Entering (13) with this value yields the first approximation x_0. Using Newton's Approximation (14) yields for the third approximation, x_2. Division of x_2 by h and squaring gives the desired quantile, $t = 0.84077663789 \times 10^{-9}$. This quantile produces a probability of $0.10000569989 \times 10^{-2}$.

While the quantile just obtained may be quite satisfactory for practical

reasons, it may be desirable to obtain an even better estimate of the specified quantile. Let us then use the quantile obtained as an initial estimate and invoke the use of Newton's Approximation in connection with the gamma distribution function. Recalling (14), i.e.,

$$x_i = x_{i-1} - (f(x_{i-1})/f'(x_{i-1})) \quad (i = 1,\ldots,m) , \tag{14}$$

let $f(x) = [F(t;\gamma) - P_g(t)]$ where $P_g(t)$ represents the probability for which we desire a quantile and $F(t;\gamma)$ denotes the computed probability for a quantile given the value of the shape parameter γ. Now $f'(x_i) = F(t;\gamma)$ where $F(t;\gamma)$ is given by (6). The derivative of $F(t;\gamma)$ is then the gamma density function, i.e.,

$$F'(t;\gamma) = (\Gamma(\gamma))^{-1}\, t^{\gamma-1}e^{-t} . \tag{19}$$

Recalling that the recurrence formula for the gamma function is $\Gamma(\gamma+1) = \gamma\Gamma(\gamma)$, then (19) may be rewritten as

$$F'(t;\gamma) = \gamma(\Gamma(\gamma+1))^{-1}\, t^{\gamma-1}e^{-t} . \tag{20}$$

The gamma function $\Gamma(\gamma+1)$ may be evaluated using an approximation by Hastings (1955):

$$\Gamma(\gamma+1) = 1 + b_1\gamma + b_2\gamma^2 + b_3\gamma^3 + b_4\gamma^4 + b_5\gamma^5 + b_6\gamma^6 + b_7\gamma^7 + b_8\gamma^8 + \varepsilon(\gamma), \tag{21}$$

where $0 \le \gamma \le 1$ and $|\varepsilon(\gamma)| \le 3\text{x}10^{-7}$.

The constants are

$b_1 = -0.577191652$	$b_5 = -0.756704078$
$b_2 = +0.988205891$	$b_6 = +0.482199394$
$b_3 = -0.897056937$	$b_7 = -0.193527818$
$b_4 = +0.918206857$	$b_8 = +0.035868343$

When $\gamma>1$, then the recurrence formula must be used to reduce the argument $(\gamma+1)$ to a value $1 \le (\gamma+1) \le 2$.

The evaluation of (6) can be done using Pearson's Expansion (1957) which may be obtained by integrating (6) by parts. The expansion is

$$F(t;\gamma) = (\Gamma(\gamma+1))^{-1}t^{\gamma}e^{-t}[1 + t/(\gamma+1) + t^2/(\gamma+1)(\gamma+2) + \ldots]. \tag{22}$$

Consider again the calculation of the quantile for a probability $P_g(t) = 0.001$ given $\gamma = 0.336$. The value of the required quantile was

$t = 0.840776663789 \times 10^{-9}$ which yields a probability $0.100000569989 \times 10^{-2}$. If, however, this quantile is used as an initial estimate and used in Newton's Approximation, a better estimate is obtained. The new estimate becomes $t = 0.840762399419 \times 10^{-9}$ which will yield a probability $0.999999999380 \times 10^{-3}$. While variations in h in the third decimal place resulted in differences in computed quantiles in the second significant digit, application of Newton's Approximation produced quantiles which differed only in the sixth significant digit. In another instance, the output from Newton's Approximation yielded results which differed only in the ninth significant digit.

For those combinations of $P_g(t)$ and γ for which $[P_g(t)]^{1/2\gamma} < 0.5 \times 10^{-10}$, the inverse of expression (22) with some modifications was used to compute quantiles. To simplify expression (22), let $t \leq 10^{-8}$ and $\gamma \leq 0.1$. The sum of the terms in brackets will yield (for practical purposes) a value of 1. Also, with $e^{-t} \simeq 1$, expression (22) becomes

$$F(t;\gamma) = (\Gamma(\gamma+1))^{-1} t^{\gamma} \tag{23}$$

Inverting (23), where F is estimated by P, yields

$$t = e^{(\gamma)^{-1} \ln[(P)(\Gamma(\gamma+1))]} \tag{24}$$

The results obtained from this expression are compatible with those given by Wilk, Gnanadesikan, and Huyett (1962).

7. Comments

The h values provided in this paper will yield quantiles which will give probabilities with a relative error of less than 0.05 percent for the selected probabilities with few exceptions. Interpolated h values also yield similar relative errors.

For probabilities other than those provided, h values may be estimated from the tables and used in computing an initial estimate of the desired quantile. Newton's Approximation then may be applied to obtain the required accuracy.

VII. PROCEDURES FOR LARGE SHAPE PARAMETERS

This section might appropriately be entitled "A New Approach to Quantile Determination." It documents a new technique for the computation of quantiles for the gamma distribution when the shape parameters are large in value, in particular, quantiles for $3 \leq \gamma \leq 104$ and 45 probabilities. The determination of quantiles for the gamma distribution has presented numerous difficulties. Most of these have resulted from the inability to invert the gamma distribution function in a direct manner. A technique was initially developed in section VI which permits quantile computation when the shape parameter is small, i.e., $\gamma < 1$. This permits a significant reduction in computer processing time for programs requiring quantile computation. This technique was later extended to shape parameters less than 4 and is included in this program.

The following technique follows in part from a procedure given by Wilk et al. (1962), but the quantile determination presented here is done in an entirely different manner. The same procedure used by Wilk et al. (1962) has also been used by Mooley (1973).

We desire to determine τ in the following expression given the shape parameter γ. (Note that the standardized form of the gamma distribution function is used to simplify computations.)

$$P(\tau) = \frac{1}{\Gamma(\gamma)} \int_0^\tau t^{\gamma-1} e^{-t}\, dt \tag{25}$$

Let $\tau v = t$, then, $dt = \tau dv$. Substituting in (25) gives

$$P(\tau) = \frac{1}{\Gamma(\gamma)} \int_0^1 \tau^{\gamma-1} v^{\gamma-1} e^{-\tau v}\, \tau dv \quad , \tag{26}$$

where v is chosen at the upper limit such that $\tau = t$, i.e., $v = 1$.
One of the properties of definite integrals is that if $g(x) \leq h(x)$ within

the interval $c \leq x \leq d$, then $\int_c^d g(x)dx \leq \int_c^d h(x)dx$. Let us apply this property to (26) to arrive at an expression which will give, upon integration, a means of estimating the lower limit of the desired quantile. By the given property, if $(v^{\gamma-1}e^{-\tau v}) \leq (v^{\gamma-1})$ where $0 \leq v \leq 1$, then we have (writing the complete expression 26)

$$P(\tau) = \frac{1}{\Gamma(\gamma)} \int_0^1 \tau^\gamma v^{\gamma-1} e^{-\tau v} dv \leq \frac{1}{\Gamma(\gamma)} \int_0^1 \tau^\gamma v^{\gamma-1} dv \tag{27}$$

$$\text{or} \quad P(\tau) \leq \frac{\tau^\gamma}{\Gamma(\gamma+1)} \;. \tag{28}$$

Solving (28) for τ gives a lower limit then for the quantile corresponding to the probability $P(\tau)$:

$$\tau = [P(\tau) \cdot \Gamma(\gamma+1)]^{1/\gamma} . \tag{29}$$

This expression is best evaluated using the logarithm of the gamma function since $\Gamma(\gamma+1)$ becomes extremely large as γ increases. The logarithm of (29), with $\Gamma(\gamma+1) = \gamma\Gamma(\gamma)$, gives

$$\ln \tau = \frac{1}{\gamma} [\ln P(\tau) + \ln \Gamma(\gamma) + \ln \gamma] . \tag{30}$$

After substitution of appropriate quantities in the right side of (30), subsequent exponentiation gives the desired lower limit of the specified quantile,

$$\tau = \exp \left[\frac{1}{\gamma} [\ln P(\tau) + \ln \Gamma(\gamma) + \ln \gamma]\right] . \tag{31}$$

The computation of $\ln \Gamma(\gamma)$ and $P(\tau)$ will be covered later on. The computation of $P(\tau)$ is required in using Newton's Approximation when checking the convergence to the desired accuracy in the quantiles.

It was in the course of working with the lower limit that the notion was conceived that a definite relationship might exist between the lower limit and the "true" or "exact" quantile. Indeed, comparison between known quantiles and lower limits indicated that a functional relationship existed. While a functional relationship has not yet been determined, an algorithm has been found which yields the ratio between the true quantile and the

lower limit (Jalickee et al., 1975). With the ratio for a selected probability and shape parameter, the true quantile may be easily determined by multiplying the computed lower limit by the ratio. If the desired accuracy is not obtained, this quantile may be subsequently used in conjunction with Newton's Approximation to provide a better estimate of the true quantile. These ratios have been called "gamma quantile ratios."

The input data for the development of the algorithm consisted of 4,725 ratios generated by a program using a Hewlett-Packard HP9830 computer. Some overlap occurred in the γ = 20 - 22 range to provide continuity for both γ-probability fields for which the algorithm was desired. Program execution began with a ratio estimate for the lowest γ-probability combination for the field being desired, i.e., γ = 3 or 20 and probability = 0.001. Once the program produced the quantile to the desired accuracy, the ratio was computed, i.e., the ratio between the true quantile and the lower limit. This ratio was not only stored in an array but also used to generate the next desired quantile. Computations of this nature continued until the array was filled. These data were stored on tape for reformatting and retrieval at a later date.

The algorithm developed uses 975 values counting the overlap mentioned above. For the field defined by γ = 3 - 22 and probability = 0.001 - 0.999, there are two arrays (5 x 45 and 5 x 20); for γ = 20 - 104 and probability = 0.001 - 0.999, the arrays become 5 x 45 and 5 x 85. The algorithm is of the form,

$$f(x,y) = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 + x_5y_5 , \qquad (32)$$

where the subscripts denote the column from which the x and y are taken. Going beyond five terms showed no significant improvement in the computed value. The algorithm input values are given in tables 4.1 and 4.2 of appendix 1. An example of the use of the algorithm is provided in appendix 2.

If it is known that quantiles computed do not have the desired level of accuracy, it becomes necessary to employ an approximation technique which hopefully does not consume too much computer time. The method employed in previous programs, and found to be applicable here with slight modifications, is Newton's Approximation Technique. Convergence using this approximation

is quadratic, a most favorable characteristic. Quadratic convergence indicates that each error is roughly proportional to the square of the error in the previous approximation. This means that the number of correct digits nearly doubles with each iteration. This popular algorithm takes the form,

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} \; , \tag{33}$$

where x_n denotes the n^{th} approximation to a root of $f(x) = 0$. In the quotient term, the $f(x_{n-1})$ denotes the function for which the root is desired but evaluated using the $(n-1)^{th}$ approximation; the prime in the denominator denotes the first derivative of the function evaluated using the $(n-1)^{th}$ approximation. Iterations are continued until the required degree of accuracy is obtained. To accommodate extremely large numbers which were observed to halt program execution during the course of quantile extraction, a slight change in (33) was required yielding

$$x_n = x_{n-1} - \exp[\ln f(x_{n-1}) - \ln f'(x_{n-1})] \; . \tag{34}$$

Now let us consider the terms in (34) which are in brackets. Define $f(x_{n-1})$ as

$$f(x_{n-1}) = F(x_{n-1}) - P \tag{35}$$

and the derivative becomes

$$f'(x_{n-1}) = F'(x_{n-1}) \tag{36}$$

since P is a constant, i.e., a preselected probability value. In general, $F(x)$ is the gamma distribution probability function so that the first derivative $F'(x)$ is the probability density function.

In effect we have, within the brackets of (34), the logarithm of the difference between the probability of the quantile approximation and the desired probability P for which the quantile is required less the logarithm of the gamma density function. The exponentiation of the term in brackets gives a correction to the $(n-1)^{th}$ approximation. The one problem which arises in using the form presented in (34) is that concerning logarithms of negative quantities. This can, however, be circumvented by using absolute values of the differences in probabilities and adding (subtracting) the corrective term when there is an underestimate (overestimate).

The evaluation of the probability F(x) required in (35) is accomplished using Pearson's Series which may be derived easily by integration of (25) by parts. Although the convergence of the series is slow, it has been used extensively. A technique is available for the evaluation of the incomplete gamma function when the shape parameter $\gamma \geq 50$ (Fisher, 1973). The accuracy of this algorithm increases with an increase in shape parameter with a subsequent decrease in the number of terms in the approximation. In essence, the incomplete gamma function is approximated by a finite series expansion -- an Edgeworth series. Overflow and rounding errors are avoided to a great extent using this method. The execution time of this algorithm is virtually independent of γ and the quantile (upper limit). Other algorithms of the type of Bhattacharjee (1970) have an execution time which, for a fixed γ value, is a bell-shaped function of the upper limit of integration. At some later time, this algorithm could be inserted into existing programs if savings in program execution times could be seen.

The form of Pearson's Series which can be used is given in its full form below. For the original form, reference may be made to several papers (Pearson, 1957; Thom, 1968). The probability $F(\tau)$ is given by

$$F(\tau) = \left[\exp(\gamma \ln \tau - \tau - \ln\Gamma(\gamma) - \ln\gamma)\right] \left[1 + \sum_{n=1}^{\infty} \exp\left(n \ln \tau - \sum_{m=1}^{n} \ln(\gamma+m)\right)\right] . \quad (37)$$

Computation of this series may be terminated when, for any n, $\exp\left(n\ln \tau - \sum_{m=1}^{n} \ln (\gamma+m)\right) \leq 10^{-6}$. In (35), when $F(\tau) - P < 10^{-9}$, computation using Newton's Method may be halted. The density function required in (33) is used in the logarithmic form in (34), i.e.,

$$\ln f(\tau) = -\tau + (\gamma-1) \cdot \ln\tau - \ln\Gamma(\gamma) . \quad (38)$$

There remains now the evaluation of $\ln \Gamma(\gamma)$ which appears in several places. Once the value is determined, it may be stored for further use. Although the expression is used in its nested form for the computer evaluation, the logarithmic form of Stirling's Series as used by Mooley (1974) is given here by

$$\ln \Gamma(\gamma) \simeq (\gamma - 0.5) \ln \gamma - \gamma + 0.5 \ln(2\pi) + \frac{A_1}{2\gamma} - \frac{A_2}{12\gamma^3} + \frac{A_3}{30\gamma^5}$$

$$- \frac{A_4}{56\gamma^7} + \frac{A_5}{90\gamma^9} - \frac{A_6}{132\gamma^{11}} + \frac{A_7}{182\gamma^{13}} - \frac{A_8}{240\gamma^{15}} , \qquad (39)$$

where the Bernoulli numbers A_1 through A_8 are $A_1 = 1/6$, $A_2 = 1/30$, $A_3 = 1/42$, $A_4 = 1/30$, $A_5 = 5/66$, $A_6 = 691/2730$, $A_7 = 7/6$, and $A_8 = 3617/510$ Mooley (1974) reports that, for $\gamma \geq 4$, an accuracy of 11 decimal places can be obtained.

To summarize, the following are the steps in determining a quantile:

1. Compute the lower limit of the quantile.
2. Use the ratio algorithm to compute the quantile ratio.
3. Multiply the lower limit by the ratio to obtain the desired quantile.
4. If the desired quantile is for other than one of the integral gamma values or the selected probability values, use Newton's Approximation to determine the quantile. In this case, use the ratio for the gamma value and probability on the lower side of the tabled combinations.

VIII. MIXED DISTRIBUTIONS

Some data sets form a mixed set of distributions. The simplest mixed set consists of two subsets of data:

1. All data equal to or less than α, the origin.
2. All data greater than α.

Where the origin α is zero, the mixed set consists of

1. The subset of zeros and
2. The subset of measured quantities.

Thus, after Thom (1951),

$$H(x) = q + p\ G(x)\ , \tag{40}$$

where q is the zero-set empirical probability, p is the measured-set probability, and G(x) is the gamma distribution function for the measured set. For example if $q = 0.40$ and $p = 0.60$, 40% of the observed values are zero and 60% of the observed values are greater than zero. Then, the cumulative probabilities of amounts greater than zero develop from the solution of G(x). These probabilities then are multiplied by 0.60 and added cumulatively to the initial 0.40 probability for the zero. If α is not a zero, then the $q = 0.40$ would apply to values $\leq \alpha$. Also, p refers to values $> \alpha$.

The above procedure, utilizing the simplest mixed distribution, is part of the present computer program. Neither the model nor the program considers or allows for mixtures within the set of measurable quantities.

IX. GAMMA DISTRIBUTION FUNCTION COMPUTER PROGRAM

Elderton (1953) provides the moment estimate procedures for the origin parameter α as indicated previously. Thom (1958, 1968) provides the requisite information and equations to provide the maximum likelihood (ML) and Thom estimates of the scale and shape parameters $\overset{*}{\beta}$ and $\overset{*}{\gamma}$.

The computer program given in appendix 3 initially follows after Bark and Hofman (1960). Since that time and through much usage, discussions, and changes, resemblance to the original program decreases. The previous program (Crutcher et al., 1973) may provide inadequate approximation for values of the probabilities when the shape parameter γ is less than 0.50. In this region, the asymptotic portion of the gamma function distribution, the slope of the curve, is almost indeterminant. Small changes in the shape parameter cause extreme changes in the function. Pearson (1922) discusses this problem. Where computers of extremely large capacity are available, the approximations may succeed at low gamma and low probability values, though numbers as small as 10^{-35} are reached before failure. When dealing with real data, such low gammas and low probabilities are not of too great importance. However, in terms of reliability problems, these may be important. Therefore, the present paper presents work done on this problem in the development of approximation algorithms or techniques. Caution is still needed when using this program for shape parameter values < 0.10.

The FORTRAN IV computer program that forms appendix 3 employs the Univac Series 70/45 computer. Use with any other computer may require a few changes, but these will be minimal. Other options may be inserted, and changes may be made by the user to satisfy his particular requirements.

Figure 2 illustrates in tabular output form the application of the gamma model to the weekly rainfall distribution at Auburn, Alabama. The first week of the climatological year, March 1-7, for 36 years with measured precipitation amounts in 40 of the years constitutes the data set. Figure 3 depicts the statistics for the second week of the climatological year, March 8-14, for 35 measured precipitation amounts in 40 years. Only 18 output levels are selected arbitrarily, though the program provides for a

AUBURN, ALABAMA PRECIPITATION PROBABILITIES 1930-1969

A 1	2	3	4	5	6	7	8	9	10	11	12
STATION	I	J	NX	NNX	XBAR	ALPHA	BETA	GAMMA	X2	PROB	K-S
10422	1	1	36	40	1.831	0.000	1.202	1.523	4.556	0.286	0.061

B 1	2	3	4	5	6	7	8	9	10	11	12	13
SEQ	ENTRY DATA	ORDER DATA	DATA /BETA	EMP PROB	EMP PROB QUANTILE B=1	EMP PROB QUANTILE B=BETA	SELECTED PROB VALUES	SELECTED QUANTILE B=1	SELECTED QUANTILE B=BETA	GRAPH PROB (X>0)	SELECTED QUANTITY LEVELS	EXC PRB FOR PCP LVL
1	4.32	0.00	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000	0.900
2	0.48	0.00	0.000	0.000	0.000	0.000	0.050	0.000	0.000	0.007	0.060	0.893
3	0.78	0.00	0.000	0.000	0.000	0.000	0.100	0.000	0.000	0.016	0.100	0.886
4	0.48	0.00	0.000	0.000	0.000	0.000	0.200	0.326	0.392	0.044	0.200	0.861
5	2.40	0.08	0.067	0.016	0.082	0.098	0.250	0.446	0.535	0.114	0.400	0.798
6	3.98	0.15	0.125	0.043	0.165	0.198	0.300	0.562	0.676	0.192	0.600	0.727
7	0.00	0.29	0.241	0.071	0.235	0.282	0.350	0.678	0.814	0.270	0.800	0.657
8	0.00	0.40	0.333	0.099	0.298	0.358	0.400	0.798	0.959	0.347	1.000	0.588
9	0.00	0.48	0.399	0.126	0.358	0.431	0.500	1.057	1.271	0.418	1.200	0.523
10	1.02	0.48	0.399	0.154	0.418	0.503	0.600	1.356	1.630	0.485	1.400	0.464
11	0.40	0.52	0.433	0.182	0.476	0.572	0.650	1.532	1.841	0.545	1.600	0.409
12	2.41	0.54	0.449	0.209	0.535	0.643	0.700	1.723	2.071	0.649	2.000	0.316
13	2.52	0.70	0.582	0.237	0.592	0.711	0.750	1.948	2.341	0.796	2.800	0.183
14	4.42	0.72	0.599	0.265	0.652	0.783	0.800	2.197	2.640	0.914	4.000	0.078
15	0.29	0.78	0.649	0.292	0.710	0.853	0.850	2.526	3.036	0.981	6.000	0.018
16	1.56	0.78	0.649	0.320	0.771	0.927	0.900	3.021	3.631	0.996	8.000	0.004
17	1.22	0.91	0.757	0.348	0.831	0.999	0.950	3.765	4.525	0.999	10.000	0.001
18	0.70	1.02	0.849	0.375	0.892	1.072	0.990	5.512	6.624	1.000	12.000	0.000
19	5.31	1.20	0.998	0.403	0.958	1.152						
20	0.52	1.22	1.015	0.431	1.023	1.229						
21	3.09	1.31	1.090	0.458	1.094	1.314						
22	0.15	1.34	1.115	0.486	1.162	1.397						
23	2.03	1.44	1.198	0.514	1.239	1.490						
24	0.72	1.56	1.298	0.542	1.314	1.579						
25	0.08	1.87	1.556	0.569	1.398	1.681						
26	0.54	2.03	1.689	0.597	1.483	1.783						
27	1.31	2.20	1.831	0.625	1.575	1.893						
28	1.20	2.24	1.864	0.652	1.667	2.004						
29	1.87	2.40	1.997	0.680	1.764	2.120						
30	2.90	2.41	2.005	0.708	1.882	2.262						
31	0.91	2.52	2.097	0.735	1.995	2.397						
32	2.24	2.90	2.413	0.763	2.136	2.567						
33	1.44	2.95	2.455	0.791	2.272	2.731						
34	3.31	3.09	2.571	0.818	2.451	2.946						
35	2.95	3.31	2.754	0.846	2.625	3.155						
36	2.20	3.98	3.312	0.874	2.872	3.451						
37	4.03	4.03	3.353	0.901	3.170	3.810						
38	0.78	4.32	3.595	0.929	3.519	4.229						
39	0.00	4.42	3.678	0.957	4.071	4.893						
40	1.34	5.31	4.418	0.984	5.189	6.237						

Figure 2. Precipitation probabilities for Auburn, AL, during the first week of the climatological year . March 1–7, 1930–1969.

AUBURN, ALABAMA PRECIPITATION PROBABILITIES 1930-1969

A 1	2	3	4	5	6	7	8	9	10	11	12
STATION	I	J	NX	NNX	XBAR	ALPHA	BETA	GAMMA	X2	PROB	K-S
10422	2	1	35	40	1.225	0.000	1.003	1.221	7.571	0.628	0.057

B 1 SEQ	2 ENTRY DATA	3 ORDER DATA	4 DATA /BETA	5 EMP PROB	6 EMP PROB QUANTILE B=1	7 EMP PROB QUANTILE B=BETA	8 SELECTED PROB VALUES	9 SELECTED QUANTILE B=1	10 SELECTED QUANTILE B=BETA	11 GRAPH PROB (X>0)	12 SELECTED QUANTITY LEVELS	13 EXC PRB FOR PCP LVL
1	0.18	0.00	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000	0.875
2	0.00	0.00	0.000	0.000	0.000	0.000	0.050	0.000	0.000	0.028	0.060	0.851
3	0.11	0.00	0.000	0.000	0.000	0.000	0.100	0.000	0.000	0.051	0.100	0.830
4	0.00	0.00	0.000	0.000	0.000	0.000	0.200	0.157	0.157	0.112	0.200	0.777
5	0.25	0.00	0.000	0.000	0.000	0.000	0.250	0.248	0.249	0.236	0.400	0.669
6	1.64	0.11	0.110	0.016	0.038	0.038	0.300	0.340	0.341	0.350	0.600	0.569
7	0.15	0.13	0.130	0.044	0.089	0.089	0.350	0.435	0.436	0.450	0.800	0.481
8	0.31	0.15	0.150	0.073	0.136	0.136	0.400	0.534	0.536	0.537	1.000	0.405
9	0.13	0.15	0.150	0.101	0.182	0.182	0.500	0.752	0.754	0.611	1.200	0.340
10	0.41	0.18	0.179	0.130	0.227	0.228	0.600	1.012	1.015	0.675	1.400	0.285
11	4.66	0.25	0.249	0.158	0.273	0.273	0.650	1.164	1.167	0.728	1.600	0.238
12	0.45	0.28	0.279	0.187	0.318	0.319	0.700	1.338	1.342	0.811	2.000	0.165
13	1.39	0.31	0.309	0.215	0.365	0.366	0.750	1.542	1.547	0.910	2.800	0.079
14	0.28	0.33	0.329	0.244	0.412	0.413	0.800	1.790	1.796	0.971	4.000	0.025
15	0.40	0.40	0.399	0.272	0.461	0.462	0.850	2.092	2.099	0.996	6.000	0.004
16	0.54	0.41	0.409	0.301	0.510	0.511	0.900	2.526	2.534	0.999	8.000	0.001
17	1.51	0.45	0.449	0.329	0.560	0.562	0.950	3.264	3.274	1.000	10.000	0.000
18	2.85	0.54	0.538	0.358	0.614	0.616	0.990	4.949	4.964	1.000	12.000	0.000
19	0.56	0.56	0.558	0.386	0.667	0.669						
20	0.61	0.61	0.608	0.415	0.725	0.727						
21	1.15	0.75	0.748	0.443	0.783	0.785						
22	0.75	0.88	0.877	0.472	0.846	0.848						
23	1.78	0.91	0.907	0.500	0.909	0.912						
24	1.26	0.91	0.907	0.528	0.975	0.978						
25	0.00	1.03	1.027	0.557	1.048	1.052						
26	0.00	1.15	1.147	0.585	1.122	1.126						
27	1.71	1.26	1.256	0.614	1.205	1.209						
28	1.54	1.26	1.256	0.642	1.289	1.293						
29	3.45	1.39	1.386	0.671	1.385	1.390						
30	1.03	1.51	1.505	0.699	1.484	1.488						
31	1.26	1.54	1.535	0.728	1.590	1.595						
32	0.91	1.64	1.635	0.756	1.717	1.722						
33	3.13	1.69	1.685	0.785	1.848	1.853						
34	0.88	1.71	1.705	0.813	2.010	2.016						
35	0.91	1.78	1.775	0.842	2.183	2.189						
36	1.69	2.85	2.841	0.870	2.408	2.416						
37	0.33	3.13	3.121	0.899	2.662	2.670						
38	0.00	3.45	3.440	0.927	3.012	3.021						
39	4.51	4.51	4.497	0.956	3.549	3.560						
40	0.15	4.66	4.646	0.984	4.634	4.648						

THE FOLLOWING CONTROL PARAMETERS HAVE BEEN READ

BEGINNING PERIOD NUMBER IS	1
ENDING PERIOD NUMBER IS	2
NUMBER OF PRECIP AND/OR PROB LEVELS IS	18
OPTION IS	1
TWO PERIOD TOTALS REQUIRED, YES IF NON-ZERO	0
THREE PERIOD TOTALS REQUIRED, YES IF NON-ZERO	0
USE DEFINED TABLES OF PRECIP OR PROB, YES IF 0	1
NUMBER OF YEARS USED IS	40
C VALUE USED IS	0.44
ALPHA VALUE TO BE COMPUTED IF IA=2,	0
VALUE TO CONTROL ITERATION LIMIT =	50
COM GRAPHS REQUESTED IF IPLOT=1	1

Figure 3. Precipitation probabilities for Auburn, AL, during the second week of the climatological year. March 8–14, 1930–1969.

maximum of 52. Fifty-two is also the maximum data set input. This latter restriction, of course, may be bypassed if the option starting with known estimates of the scale and shape parameters is used. Parts A and B in figures 2 and 3 and in data output divide the tabulations into two sets of columns. Part A, shown on the first line, provides the following:

1	Station	Identification
2	I	Sample number
3	J	Number of duration periods in sample
4	NX	Number of data excluding zeros
5	NNX	Number of data including zeros
6	XBAR	Arithmetic average of data excluding zeros
7	ALPHA	Origin value
8	BETA	Scale parameter estimate, BETA STAR
9	GAMMA	Shape parameter estimate, GAMMA STAR
10	X2	X^2 for chi-square test
11	PROB	Probability of a chi-square equal to X^2 above
12	K-S	The largest difference in probability between the theoretical and empirical distribution curves. This is the Kolmogorov-Smirnov test statistic (Smirnov, 1948).

Part B comprises 13 columns of output information that provide the following:

1 Sequential guidance

2 Data in order of observation or record. These are x or y or transforms of y such as $(y-\alpha)$ or $(y-\alpha)/\beta$.

3 Ordered data of column 2

4 Ordered data of column 2 divided by the scale parameter β of column B-2 data. If the transform $((y-\alpha)/\beta)$ is used, columns B-2 and B-4 ought to be identical except for rounding error.

5 Empirical probability of the ordered data. The expression $(n-c)/(n-c+1)$ provides the probabilities where n is equivalent to NX of part A and c = 0.44 (Gringorten, 1963). NX is the number of nonzero data. A program option permits a change in the value of c.

6 Variate quantile associated with the empirical probability of column 5 with the scale parameter β set equal to unity.

7 Variate quantile associated with the empirical probability of column 5 with the sample scale parameter $\overset{*}{\beta}$ (beta star) shown in part A.

8 Fifty-two or less arbitrarily selected cumulative theoretical probability values for which columns B-9 and B-10 respectively show corresponding cumulative quantiles and amounts. A program option permits change in these, but allows for no more than 52.

9 Cumulative quantile values of the distribution corresponding respectively to the cumulative probability values of column B-8.

10 Cumulative values of the distribution corresponding respectively to the cumulative probability values of column B-8. Multiplication of values in column B-9 by the sample $\overset{*}{\beta}$ (beta star) value of part A provides column B-10 data.

11 Consider the base. The base is only the distribution of nonzero amounts shown in columns B-2 and B-3. The number of data is the NX of column A-4. Column B-11 then gives the probabilities of occurrence of amounts equal to or less than selected nonzero amounts shown in column B-12. This column is labeled "GRAPH" to indicate that this may be used to graph the set of nonzero amounts.

12 Arbitrarily selected cumulative amounts. The maximum number of amounts is 52. A program option permits change to less than 52 amounts. The option also provides for the amounts to be scaled in terms of the mean of the nonzero amounts.

13 Probabilities of exceeding the arbitrarily selected cumulative amounts shown in column B-12. This is the mixed distribution.

If NX = NNX in part A, columns A-4 and A-5 (i.e., if the original distribution has no zero amounts), then column B-13 is the complement of column B-11.

The plot of columns B-11, B-12, and B-13 (as one set) and columns B-8 and B-9 (as another set) should plot on the straight line of the graphs shown in this report. The data of column B-12 should be scaled by division by the scale parameter β. The empirical probabilities and empirical amounts shown

in columns B-5 and B-7 plotted on the graph will show visually and subjectively how good the line of best fit fits the data.

Wherever the approximation routines fail for a particular quantity or probability level, this will be noted in the output. Usually, enough levels will be available so that the loss of a level or two is not important (i.e., interpolation will suffice). If too many levels are noted, then the program routines generally will be inadequate because of difficulties previously mentioned in the asymptotic portion of the distribution.

For most purposes (in the analytical sense), if the gamma model is accepted without question, columns B-8 and B-10 or columns B-12 and B-13 provide the desired information based on the data sample. One set is the inverse of the other, though different levels may be and generally are used.

X. GRAPHS

Graphs used here are based on the work of Crutcher, Barger, and McKay (1973). The design of figures 2 and 3 permits the easy extraction of information for manual or automatic plotting. The program prior to subroutine plot (line 1310) in appendix 3 incorporates the command instructions for the CALCOMP Plotter System to a drum computer and/or Computer Output to Microfilm (COM). An option permits selection of any or all the graphs and either or both plotters. These particular instructions can be used only if the CALCOMP computing package is available. However, if this is not available, modifications based on details given here can be used to control any x-y plotter.

Four graphs are developed here. Many others could be developed from the great amount of information contained in the computer output exemplified in figures 2 and 3. These graphs developed here permit:

(1) a visual assessment of the theoretical density based on the parameter estimates,

(2) a visual assessment of the chi-square statistics by means of a histogram,

(3) a visual assessment of the Kolmogorov-Smirnov statistics by the use of a cumulative ordered interval data overplot on the theoretical cumulative intervals; and

(4) a visual assessment of the line of best fit which has a line of slope 1, the diagonal line on the graph. An ordered data overplot is provided.

The preparation of the graphs are now discussed in the above order.

1. Density Curves

Figures 4a and 5a provide a truncated density curve for the non-zero data in figures 2 and 3, respectively. The size of the plot on microfilm is as large as the Computer Output to Microfilm (COM) permits,but the plot can be enlarged photographically to any desired size. The vertical scale maximum is standardized to 1 and divided into fourths. The maximum value is always located on the horizontal scale at a quantile computed for a shape parameter

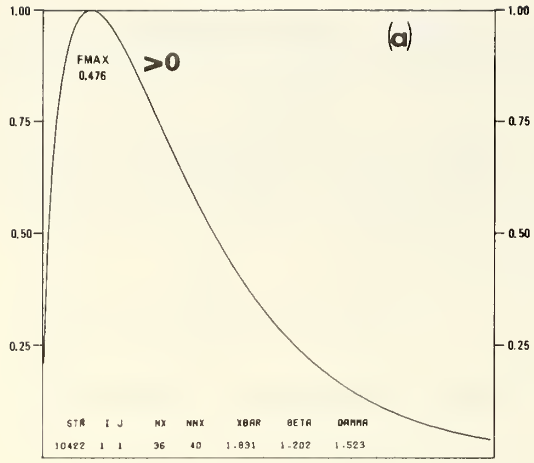


FIGURE 4a. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 1–7, 1930–1969.

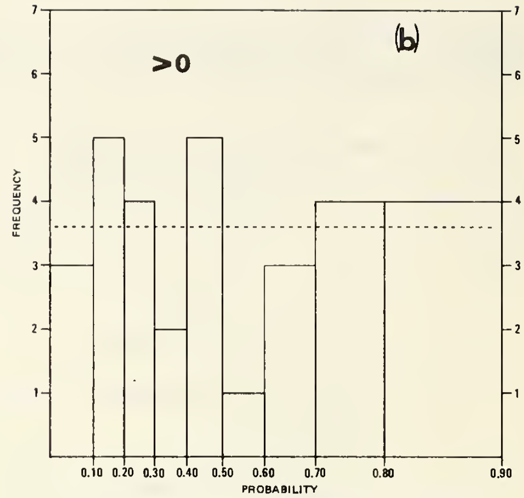


FIGURE 4b. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 1–7, 1930–1969.

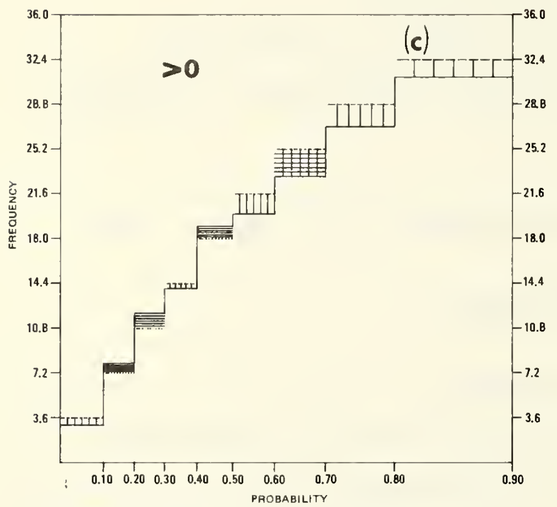


FIGURE 4c. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 1–7, 1930–1969.

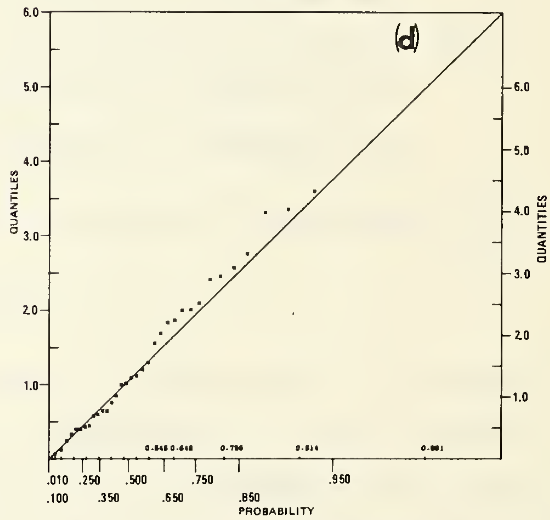


FIGURE 4d. AUBURN ALABAMA PRECIPITATION PROBABILITIES MARCH 1–7, 1930–1969.

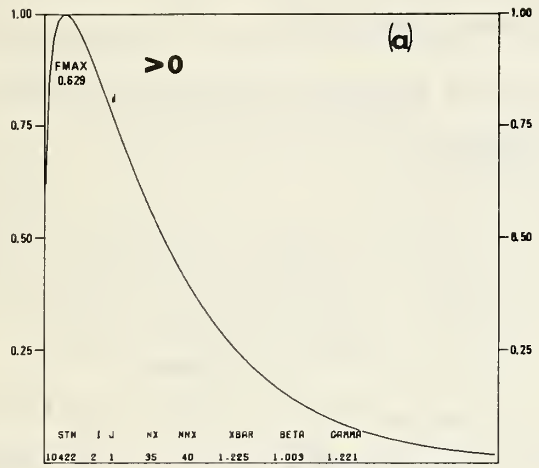


FIGURE 5a. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 8–14, 1930–1969.

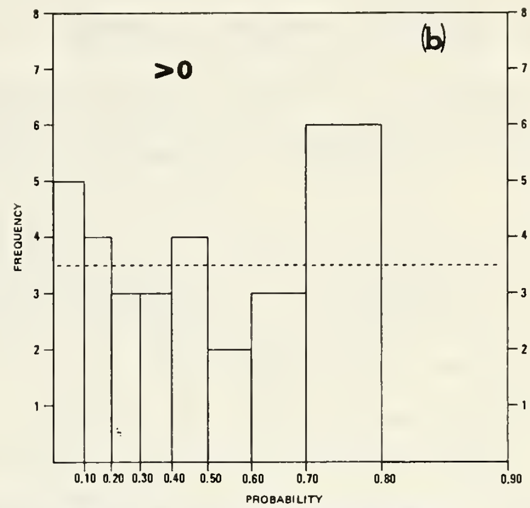


FIGURE 5b. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 8–14, 1930–1969.

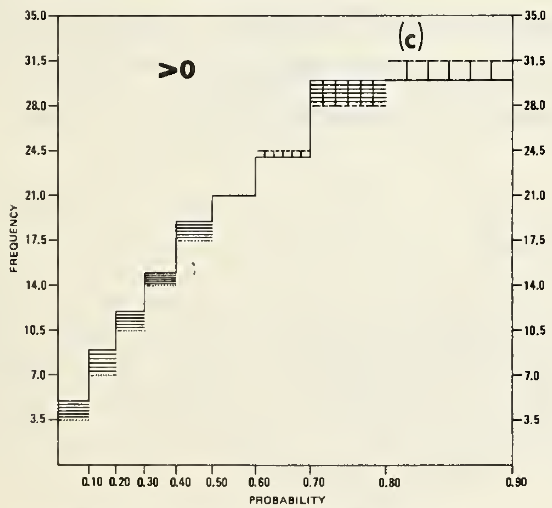


FIGURE 5c. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 8–14, 1930–1969.

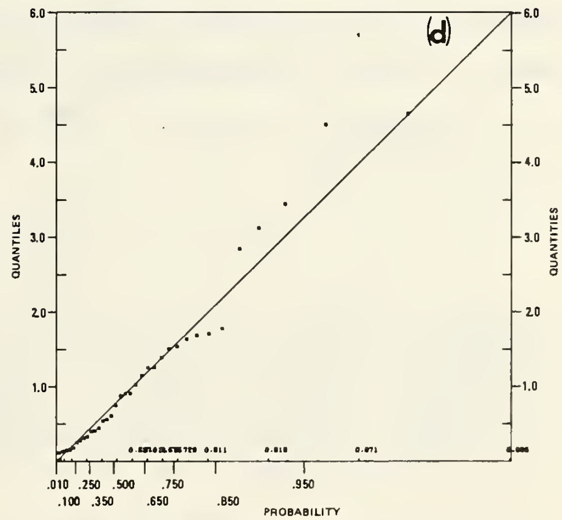


FIGURE 5d. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 8–14, 1930–1969.

value one less than the actual shape parameter estimate. That is, the modal value of a gamma distribution is always at (γ-1). When the shape parameter is 1 (the exponential distribution), the modal value is at 0. When the shape parameter is less than 1, the modal value is indeterminate. In the last case, the modal value will be truncated at the top of the square. The legend below the figures contains the modal maximum. If it is equal to or greater than 1000, computing will be stopped and the legend will contain the statement that the modal maximum is equal to or greater than 1000.

Equation (41) is the analytic expression of the density function f in terms of the quantile t and the shape parameter, γ.

$$f = f(t,\gamma) = (\Gamma(\gamma))^{-1}\ t^{\gamma-1}\ e^{-t} \qquad (41)$$

In the calculation of the X^2 statistic, an option permits the selection of any number of equal probability class intervals. If the option is not exercised, the default option is ten. It is impossible to compute the plotting quantile for 100 percent. As the number of intervals is optional, it seems best here to simply drop off the last interval, that is, to truncate the distribution at the $(k-1)^{th}$ interval. If the user is not satisfied with this, the program may be modified to permit extension to any predetermined probability level (except 1.00) or quantile. In most cases, a representative density curve will be obtained. Comparison of this density curve with the curve expressed for a given shape parameter (see figure 1) will provide a consistency check.

2. Histograms

Figures 4b and 5b, respectively, are truncated histograms for nonzero data contained in figures 2 and 3. Truncation is made here for the same reasons given above. That is, the last probability interval is not shown. This does not invalidate comparisons but some care must be exercised.

The expected frequency histogram is shown by the dashed line. Simply, it is a value equal to NNX divided by the number of intervals k, in this case to ten. Where 20 cases are used, the expected frequency in each equal probability class interval is 2.0. Where there are 25 cases, the expected frequency shown would be 2.5.

The actual frequency histogram is shown by the solid lines. It is the differences of these intervals, the expected frequencies less the observed frequencies, that are used in the calculation of the X^2 statistics (X not chi). The frequency difference is squared and then divided by the expected frequency. One such value is available for each equal probability class interval. It is the addition of all of these values that gives the value of X^2 which is shown on line A of figures 2 and 3.

The horizontal scale is plotted in terms of quantiles which are linear for the probability levels dictated by the option of class intervals. For example, 10 class intervals would select probabilities of 10, 20,....90 percent, while 20 class intervals would select probabilities of 05, 10,....95 percent. Therefore, the distances between the vertical scale of the frequency (bar) histogram will be variable and will depend on the shape parameter. The narrowest bar will be in the neighborhood of the modal point of the density curve.

These figures are plotted to permit an assessment of symmetry. If the figures imply considerable asymmetry, the use of the X^2 statistics or the chi-square test should be suspect. To begin with, the chi-square test is not a powerful test. The greater the number of data in each interval, the better the test. However, an increase of the number of class intervals does not necessarily increase the power of the test [Tate and Hyer (1973)]. For this test, a maximum of 20 equi-probability class intervals is suggested.

These histograms are quite different in appearance than those where equal class intervals in terms of the quantities or in terms of the variate are chosen. Symmetry will be indicated when there is a single horizontal line across the figure at the expected frequency. That is, the solid horizontal line will be over the dashed horizontal line. Asymmetry will be implied when the solid lines are low on the left and high on the right or high on the left and low on the right.

These figures must not be used for assessment of areas under the curve. The areas within each interval under the density curve would be equal. Therefore, it is only the vertical differences between the dashed lines and the interval tops that are to be used here.

These tests apply only to the non-zero data, i.e., the data greater than the origin.

3. Cumulative Frequency Diagrams

The cumulative diagrams, figures 4c and 5c from figures 2 and 3, respectively, permit assessment of the Kolmogorov-Smirnov statistics as discussed by Crutcher et al. (1973), Kolmogorov (1933), and Smirnov (1936, 1948). The horizontal scale is linear in terms of quantiles as in figures 4 through 6 and again is marked in time of equal-probability class intervals. The vertical scale is in terms of frequencies with a maximum value equal to the number of nonzero data, NNX of figures 2 and 3, respectively. Dashed lines provide the expected frequency cumulative diagram, while the solid lines provide the observed frequency cumulative diagram. Where they are the same, only the solid line will show. Differences are indicated by the vertical or horizontal hachuring. When the expected frequency is greater than the observed frequency, the hachuring is vertical. The greatest difference is indicated by both vertical and horizontal hachuring. Where there are ties, only one will be indicated. The greatest difference between the cumulative curves is the basis of the Kolmogorov-Smirnov (K-S) test discussed by Crutcher et al. (1973). Though there appear to be discrete data, they are from continuous distributions. Figures 2 and 3 present the K-S statistics as the last item in line A. These tests apply only to the nonzero data. Table 5 (appendix 1) provides data to test the significance of the K-S statistics [Lilliefors (1967, 1969, 1973) and Crutcher (1975)]. For example the values of the K-S statistics of figures 2 and 3, 0.061 and 0.057 for n values of 36 and 35 with gamma values between one and two, are not significant. Therefore, the null hypothesis that the gamma fit is not significantly different from the data set is not rejected. The gamma fit is then used.

4. Ordered Data Model Fit

This type of graph as presented in figures 4d and 5d from figures 2 and 3, respectively, is perhaps the most important of the four presented here. The scaling is the same in all of figures 4 and 5 in that it is linear in the quantiles. The vertical scaling is in terms of quantiles on the left and

quantities on the right. Ordinarily, the units are obtained by multiplying the quantiles by the scale parameter; but in order to scale properly for plotting, division is required. The vertical scaling on the right, marked quantities, is then in units of the data, in this case inches. The type of data, i.e., precipitation, is not placed on the scale. The scaling numbers on the right will be integer values only when the scale parameter is an integer. Plotting, as always, is in terms of the linear quantiles marked in terms of probabilities along the abscissa, and in the ordinate quantiles on the left and data units on the right vertically. The line of best fit is the 45 degree line, line of slope 1, running diagonally from the lower left to the upper right of the square. The symbols represent the relative position of the actual observed data. A visual assessment of the fit is easily made. Here, the fit to each data set appears acceptable. This decision also is supported by the non-rejection of the χ^2 and the Kolmogorov-Smirnov tests of the null hypothesis.

There are two probability scales on the horizontal axis. The tick marks extending upward on the inside refer to the probabilities for amounts greater than zero shown in columns 4 and 12 of figures 2 and 3. The tick marks extending downward refer to the probabilities for amounts equal to or greater than zero, i.e., the zeros are included. The reference columns are 8 and 10.

Figure 6 is presented as a modification of figure 4d to illustrate the relationship between figure 2 and figure 4d. Columns 12 and 11 are first chosen. The quantity level of 6 inches of precipitation and its corresponding probability of 0.981 are chosen. The quantity 6 inches is divided by the scaling factor 1.202 to obtain 4.991. The first step on the graph is to draw a horizontal line parallel to the baseline at 4.991 (on the left as a quantile) across figure 4d as in figure 6. In the second step, in units of the variable, the value of 6.00 will be read on the right. Third, extend a line vertically downward from the intersection of the diagonal line of best fit and the line drawn in step 1. Mark this with an upward extending tick from the baseline and label with the appropriate theoretical probability 0.981. This may be repeated for each of the paired values of columns 12 and 11.

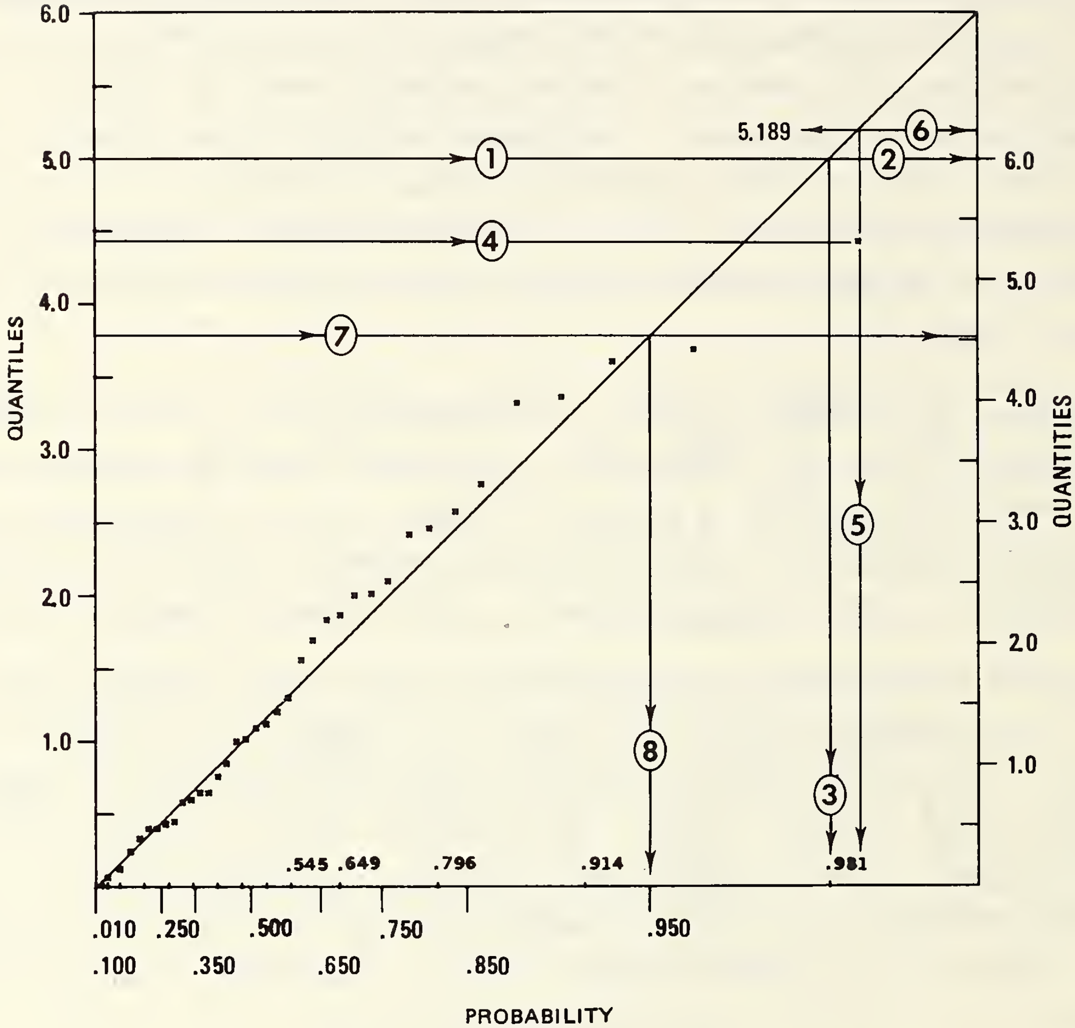


FIGURE 6. AUBURN, ALABAMA PRECIPITATION PROBABILITIES MARCH 1–7, 1930–1969.

As plotting from columns 4, 6,and 5 of figure 2 is also for amounts greater than zero, these are now plotted and compared on the same scale indicated by the upward extending tick marks inside the square. For the last point plotted on the graph, the appropriate values are 4.418, 5.189,and 0.984, respectively. As a fourth step, the quantile value of 4.418 on the vertical is plotted against the empirical quantile value of 5.189 and marked with some symbol, here a small cross (×). Through the cross, extend a line vertically downward as step 5 and mark with the appropriate value of 0.984 if required. As step 6, extend the vertical line in step 5 upward to the diagonal line of best fit and then horizontally to the units scale on the right where the appropriate quantity of 6.237 is then read or approximated.

To keep the numbers from over-printing too frequently, only the selected probabilities above 0.50 are plotted and marked although all selected probability tick marks are shown. If the user should sometimes use more probability levels, over-printing will have to be accepted or selection for printing changed in the program. Even then, when the shape parameter is small, there may be difficulty which could be avoided by change of scale or curtailment of printing.

For amounts associated with selected probability levels, the data in columns 8, 9 and 10 of figure 2 are used. At selected probability levels the appropriate amounts or less are given. For step 7, the probability level of 0.950, the quantile 3.765, and the unit value of 4.525 inches are used. A line is extended horizontally from 3.765 on the left across the diagonal line of best fit to the right-hand side where the value of 4.525 is read. As step 8, a line is extended downward to the base, marked with a tick mark extending downward and labeled as 0.950, the corresponding probability. Repeating these steps for other paired values in columns 8 and 9 will provide an appropriate probability scale. A full grid, not shown here, can be obtained by change of instructions to draw the entire lines rather than just the tick marks shown here.

XI. GRAPH PAPER

Many investigators determine a line of best fit of many models to a set of data. Then they choose the model which appears to provide the best of the many lines of best fit. This may be done in terms of a visual fit or in terms of a least squares of deviations fit. This is a dangerous policy for there are errors of the first and second kind involved. No causal relationships can be drawn in such a post analysis. If there is an "a priori" design of an experiment and a variable(s) is(are) under the control of the experimenter, then some causal or physical relationships might be established.

In all of the foregoing it has been assumed that the gamma distribution is an acceptable model. As pointed out by Crutcher, Barger, and McKay (1973), rainfall may be likened to a study of reliability or failure. They provide graph paper covering several ranges of tables provided by others such as Thom (1968), Harter (1964), Wilk, Gnanadesikan and Huyett (1962), and Pearson (1922). Tables are used because (except for the exponential distribution when the shape parameter γ equals one) when $G(y) = 1 - \exp(-y)$, the inverse form G^{-1} cannot be put in closed form. Some tables are provided in appendix 1.

At this point it is relevant to point out that, in using moment estimates, the following relationships hold. As a rough cross-check, the arithmetic mean and the sample standard deviation can be used to check the graphical estimates. The mean is equal to the product of the scale and shape parameters. The variance is equal to the shape parameter times the square of the scale parameter or is equal to the mean times the scale parameter. This allows one to quickly calculate two of the parameters when the other two are available. In symbolic form these relationships are

$$\bar{x} = \hat{\beta}\,\hat{\gamma} \qquad \text{or} \qquad \hat{\beta} = s^2/\bar{x} = \hat{\sigma}^2/\hat{\mu} \quad .$$

$$s^2 = \hat{\beta}^2\,\hat{\gamma} \qquad\qquad \hat{\gamma} = \bar{x}^2/s^2 = \hat{\mu}^2/\hat{\sigma}^2$$

$$(\text{given } \hat{\beta} \text{ and } \hat{\gamma}) \qquad\qquad (\text{given } \bar{x} \text{ and } s^2 \text{ as } \hat{\mu} \text{ and } \hat{\sigma}^2)$$

In this sense the shape parameter is equal to the reciprocal of the square of the coefficient variation.

Crutcher et al. (1973) present graph paper for the gamma distribution covering several ranges of the shape parameter. This paper also may be used to estimate the scale and shape parameters. Figure 7, modified from Kao (1968), illustrates another form of graph paper for plotting and graphical estimation of parameters. (Kao's paper was not available to the authors prior to the preparation of the 1973 paper.) The hypothetical data and procedures used by Kao and given in appendix 1 as table 6 are used here to illustrate further the application of the gamma distribution to reliability problems. Gupta and Groll (1961) provide further discussion.

Gringorten (1963) and Crutcher et al. (1973) use $(i-0.44)/(n+0.12)$ rather than $i/(n+1)$ to determine plotting positions. Blom (1958) and Kimball (1960) discuss this problem in detail.

The gamma probability paper (figure 7) has two ordinates. The right-hand one is the y-scale which is linear and the left-hand one is the p-scale which is equal to $G(y)$ for $\gamma = 1$ (exponential case). The extreme left portion of the paper gives the p-scale for other γ-values in the range of (0.5 to 5.0). In plotting the data, one chooses the p-scale by trial and error (there are infinitely many in the range) until the plot appears to be "linear." The diagram at the upper left corner reminds us of the estimates for α, the location parameter, and β, the scale parameter. Life quality characteristics such as mean life, standard deviation of life, and median life can be estimated graphically through the estimates for α and β. For each estimate for γ (the γ value which yields a linear plot), there is a corresponding estimate value for μ/β and σ/β. The additional scales on the top of the graph paper are for small values of γ. Knowing μ/β and σ/β and estimates for α and β, μ and σ can be estimated. Other life quality characteristics of interest are reliability functions at some x, and reliable life at any specified reliability index r. These quantities will now be defined and estimated graphically. The reliability function $R(x)$ is defined as $1 - F(x)$ for any x. Since the probability plot is in itself an estimate of $F(x)$, the estimate of $R(x)$ can be readily obtained by reading the $[1 - F(x)]$ value from the appropriate p-scale for any x value. The reliable life p_r is the inverse function of $R(x)$, or $p_r = R^{-1}(r)$ which is solved on the probability plot by starting

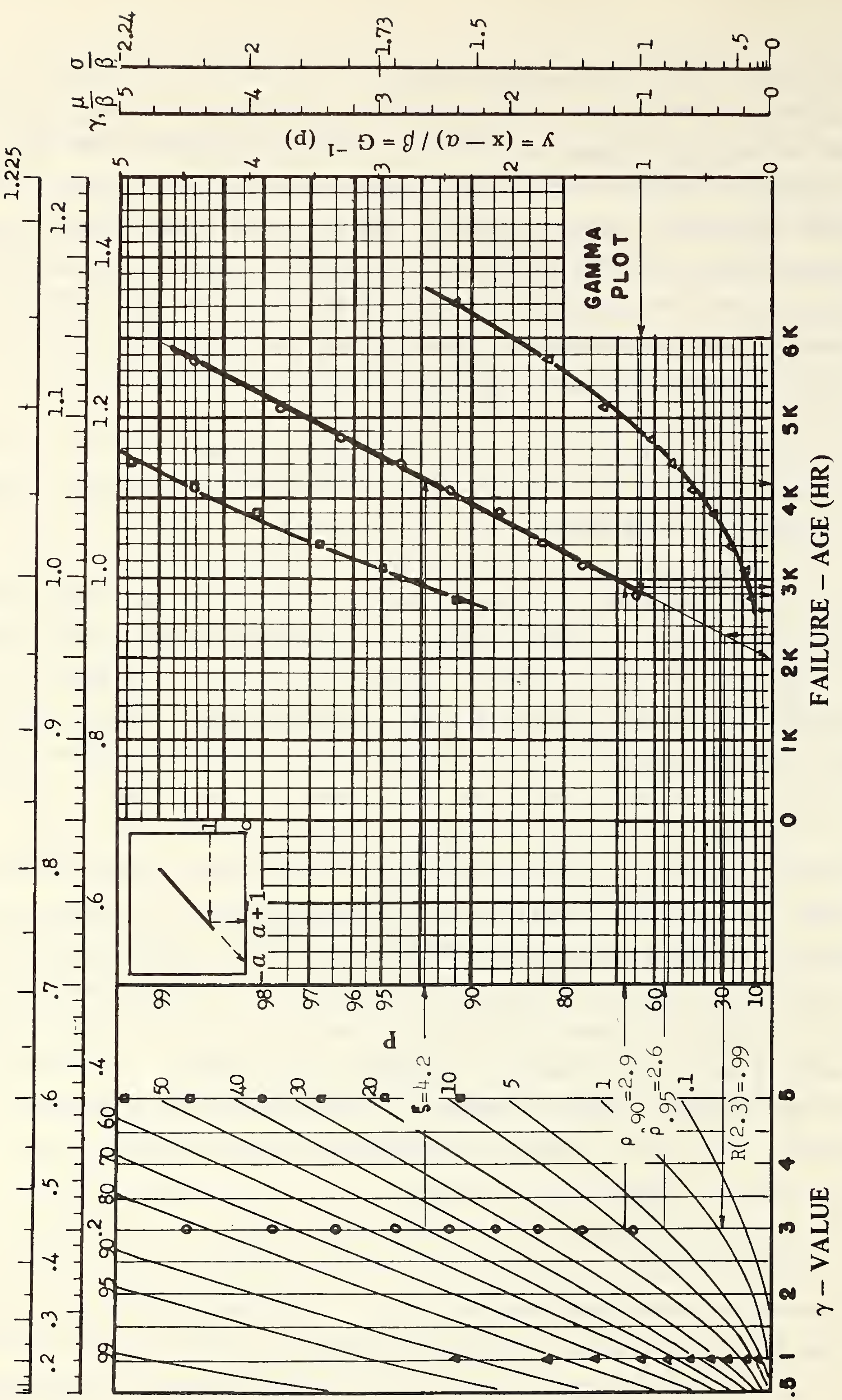


Figure 7. Gamma plot of failure-age (after Kao, 1968).

from (1 - r) value at the appropriate p-scale horizontally to the probability plot and reading p_r from the abscissa. The median life is equal to $p_{.50}$ -- a special case of the reliable life.

Figure 7 shows that three trials on the data have been made, one for each of the following γ values; $\gamma = 1$, $\gamma = 3$, and $\gamma = 5$. Apparently, $\gamma = 3$ yielded the most linear plot, with $\mu/\beta = 3$ and $\sigma/\beta = 1.73$. Hence, the estimates for the gamma parameters are

Location: $\hat{\alpha} = 2$ khr.

Scale: $\hat{\beta} = 2.8 - 2 = 0.8$ khr.

Shape: $\hat{\gamma} = 3$ (no dimension).

A few further calculations give the following estimates:

Mean = $\hat{\alpha} + \hat{\beta}(\mu/\beta) = 2 + 0.8(3) = 4.4$ khr.

Standard deviation = $\hat{\beta}(\sigma/\beta) = 0.8(1.73) = 1.38$ khr.

Reliability function at 2.3 khr. = 0.99.

Reliable life at 90% = 2.9 khr.

Reliable life at 95% = 2.6 khr.

Median life = 4.2 khr.

XII. FUTURE MODIFICATIONS TO THE PROGRAM

The following are six expected modifications planned for the computer program either as a part of the program or as separate programs:

1. A subroutine for the determination of a better theoretically and practically acceptable location (origin) parameter estimate.
2. A possible subroutine for the debiasing of the maximum likelihood and Thom (1958) shape and scale estimators.
3. Separate gamma distribution programs for one data string to provide (a) quantiles for specified probability levels and (b) probabilities for specified quantiles.
4. Separate normal distribution program for one data string for cases where the shape parameter estimate is greater than 100 to provide (a) quantiles for specified probability levels and (b) probability levels for specified quantiles.
5. The development of confidence levels (tolerance bands) for the lines of best fit. The binomial distribution has been used. This will be compared with Monte Carlo techniques.
6. The examination of possible better estimators for the parameters of the gamma distribution, particularly where all three parameters, origin, scale, and shape, are unknown.

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AUTHOR INDEX

APPENDIX 1

TABLES

Table 1

Values of h as a function of gamma (γ) and selected probabilities

P \ γ	.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
.001	1.6658	1.6084	1.5514	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2807
.003	1.6658	1.6084	1.5515	1.5010	1.4554	1.4142	1.3766	1.3422	1.3103	1.2808
.005	1.6658	1.6084	1.5515	1.5010	1.4554	1.4142	1.3766	1.3422	1.3103	1.2808
.006	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2808
.007	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2808
.008	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2808
.009	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2808
.010	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2808
.015	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2809
.020	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3103	1.2809
.025	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3104	1.2810
.030	1.6658	1.6084	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3104	1.2810
.035	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3104	1.2811
.040	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3104	1.2811
.045	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3104	1.2812
.050	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3105	1.2812
.055	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3105	1.2813
.060	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3105	1.2813
.065	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3106	1.2814
.070	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3766	1.3422	1.3106	1.2814
.075	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3767	1.3423	1.3107	1.2815
.080	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3767	1.3423	1.3107	1.2815
.085	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3767	1.3423	1.3107	1.2816
.090	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3767	1.3423	1.3108	1.2816
.095	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3767	1.3423	1.3108	1.2817
.100	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3767	1.3424	1.3108	1.2817
.150	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3768	1.3426	1.3112	1.2824
.200	1.6658	1.6083	1.5515	1.5009	1.4554	1.4142	1.3769	1.3428	1.3117	1.2830
.250	1.6658	1.6083	1.5515	1.5008	1.4553	1.4142	1.3770	1.3431	1.3122	1.2838
.300	1.6658	1.6083	1.5515	1.5008	1.4552	1.4142	1.3771	1.3435	1.3127	1.2845
.350	1.6658	1.6083	1.5514	1.5006	1.4551	1.4142	1.3773	1.3439	1.3134	1.2854
.400	1.6658	1.6083	1.5513	1.5005	1.4550	1.4142	1.3775	1.3443	1.3140	1.2863
.450	1.6658	1.6082	1.5512	1.5002	1.4548	1.4142	1.3778	1.3448	1.3148	1.2873
.500	1.6658	1.6082	1.5510	1.4999	1.4546	1.4142	1.3780	1.3453	1.3156	1.2883
.550	1.6658	1.6082	1.5507	1.4995	1.4543	1.4142	1.3783	1.3459	1.3165	1.2896
.600	1.6658	1.6081	1.5502	1.4990	1.4540	1.4142	1.3787	1.3467	1.3176	1.2909
.631	1.6658	1.6080	1.5499	1.4990	1.4538	1.4142	1.3789	1.3471	1.3183	1.2919
.650	1.6658	1.6079	1.5496	1.4984	1.4536	1.4142	1.3791	1.3475	1.3187	1.2925
.700	1.6658	1.6076	1.5487	1.4975	1.4532	1.4142	1.3796	1.3484	1.3201	1.2942
.750	1.6658	1.6069	1.5473	1.4964	1.4526	1.4142	1.3801	1.3495	1.3217	1.2962
.800	1.6658	1.6057	1.5454	1.4949	1.4518	1.4142	1.3808	1.3508	1.3236	1.2987
.850	1.6658	1.6033	1.5426	1.4929	1.4509	1.4142	1.3817	1.3525	1.3260	1.3018
.900	1.6658	1.5987	1.5382	1.4900	1.4494	1.4142	1.3830	1.3549	1.3293	1.3059
.910	1.6658	1.5973	1.5370	1.4892	1.4491	1.4142	1.3833	1.3555	1.3302	1.3070
.930	1.6658	1.5937	1.5341	1.4874	1.4482	1.4142	1.3840	1.3569	1.3321	1.3094
.950	1.6656	1.5884	1.5301	1.4849	1.4471	1.4142	1.3850	1.3587	1.3347	1.3126
.970	1.6643	1.5799	1.5242	1.4813	1.4455	1.4142	1.3864	1.3613	1.3383	1.3171
.980	1.6615	1.5732	1.5196	1.4786	1.4442	1.4142	1.3874	1.3632	1.3410	1.3205
.990	1.6522	1.5619	1.5123	1.4743	1.4423	1.4142	1.3891	1.3662	1.3452	1.3257
.995	1.6388	1.5514	1.5057	1.4703	1.4405	1.4142	1.3905	1.3689	1.3490	1.3305
.997	1.6277	1.5443	1.5012	1.4678	1.4394	1.4142	1.3915	1.3707	1.3516	1.3338
.999	1.6040	1.5309	1.4928	1.4629	1.4372	1.4142	1.3933	1.3742	1.3565	1.3398

Table 1 (cont'd)

Values of h as a function of gamma (γ) and selected probabilities

P \ γ	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
.001	1.2533	1.2277	1.2038	1.1812	1.1600	1.1401	1.1211	1.1033	1.0863	1.0702
.003	1.2534	1.2278	1.2038	1.1814	1.1602	1.1403	1.1215	1.1037	1.0868	1.0707
.005	1.2534	1.2278	1.2039	1.1815	1.1604	1.1405	1.1218	1.1040	1.0871	1.0711
.006	1.2534	1.2279	1.2040	1.1815	1.1605	1.1406	1.1219	1.1041	1.0873	1.0713
.007	1.2534	1.2279	1.2040	1.1816	1.1605	1.1407	1.1220	1.1042	1.0874	1.0715
.008	1.2534	1.2279	1.2040	1.1816	1.1606	1.1408	1.1221	1.1044	1.0876	1.0716
.009	1.2534	1.2279	1.2041	1.1817	1.1606	1.1409	1.1222	1.1045	1.0877	1.0718
.010	1.2535	1.2279	1.2041	1.1817	1.1607	1.1409	1.2222	1.1046	1.0878	1.0719
.015	1.2535	1.2281	1.2043	1.1820	1.1611	1.1413	1.1227	1.1051	1.0884	1.0725
.020	1.2536	1.2282	1.2044	1.1822	1.1613	1.1416	1.1230	1.1055	1.0889	1.0731
.025	1.2537	1.2283	1.2046	1.1824	1.1615	1.1419	1.1234	1.1059	1.0893	1.0736
.030	1.2538	1.2284	1.2047	1.1826	1.1617	1.1422	1.1237	1.1062	1.0897	1.0740
.035	1.2538	1.2285	1.2049	1.1827	1.1620	1.1424	1.1240	1.1066	1.0901	1.0745
.040	1.2539	1.2286	1.2050	1.1829	1.1622	1.1427	1.1243	1.1069	1.0905	1.0749
.045	1.2540	1.2287	1.2051	1.1831	1.1624	1.1429	1.1246	1.1072	1.0908	1.0752
.050	1.2541	1.2288	1.2053	1.1833	1.1626	1.1432	1.1248	1.1075	1.0912	1.0756
.055	1.2541	1.2289	1.2054	1.1834	1.1628	1.1434	1.1251	1.1078	1.0915	1.0760
.060	1.2542	1.2290	1.2056	1.1836	1.1630	1.1436	1.1254	1.1081	1.0918	1.0763
.065	1.2543	1.2291	1.2057	1.1838	1.1632	1.1439	1.1256	1.1084	1.0921	1.0767
.070	1.2544	1.2293	1.2058	1.1839	1.1634	1.1441	1.1259	1.1087	1.0924	1.0770
.075	1.2545	1.2294	1.2060	1.1841	1.1636	1.1443	1.1261	1.1090	1.0927	1.0773
.080	1.2545	1.2295	1.2061	1.1842	1.1638	1.1445	1.1264	1.1092	1.0930	1.0776
.085	1.2546	1.2296	1.2062	1.1844	1.1640	1.1447	1.1266	1.1095	1.0933	1.0779
.090	1.2547	1.2297	1.2064	1.1846	1.1641	1.1449	1.1268	1.1098	1.0936	1.0782
.095	1.2548	1.2298	1.2065	1.1847	1.1643	1.1451	1.1271	1.1100	1.0939	1.0785
.100	1.2549	1.2298	1.2066	1.1849	1.1645	1.1454	1.1273	1.1103	1.0941	1.0788
.150	1.2557	1.2310	1.2079	1.1864	1.1664	1.1474	1.1295	1.1127	1.0968	1.0817
.200	1.2566	1.2321	1.2093	1.1880	1.1680	1.1493	1.1317	1.1150	1.0992	1.0843
.250	1.2575	1.2332	1.2106	1.1895	1.1698	1.1513	1.1338	1.1173	1.1017	1.0869
.300	1.2585	1.2344	1.2121	1.1912	1.1716	1.1532	1.1359	1.1196	1.1041	1.0894
.350	1.2596	1.2357	1.2135	1.1928	1.1735	1.1553	1.1381	1.1219	1.1066	1.0920
.400	1.2607	1.2371	1.2151	1.1946	1.1754	1.1574	1.1404	1.1243	1.1091	1.0947
.450	1.2620	1.2386	1.2168	1.1965	1.1775	1.1596	1.1428	1.1268	1.1118	1.0975
.500	1.2633	1.2402	1.2186	1.1985	1.1797	1.1620	1.1453	1.1295	1.1146	1.1004
.550	1.2648	1.2419	1.2206	1.2007	1.1820	1.1645	1.1480	1.1324	1.1176	1.1035
.600	1.2664	1.2438	1.2227	1.2030	1.1846	1.1673	1.1509	1.1355	1.1208	1.1069
.631	1.2664	1.2451	1.2241	1.2046	1.1863	1.1691	1.1529	1.1375	1.1230	1.1091
.650	1.2683	1.2459	1.2251	1.2057	1.1874	1.1703	1.1541	1.1389	1.1244	1.1106
.700	1.2704	1.2483	1.2278	1.2086	1.1906	1.1737	1.1578	1.1427	1.1283	1.1147
.750	1.2728	1.2511	1.2309	1.2120	1.1943	1.1776	1.1619	1.1470	1.1329	1.1194
.800	1.2757	1.2544	1.2346	1.2161	1.1987	1.1823	1.1668	1.1521	1.1382	1.1250
.850	1.2793	1.2586	1.2392	1.2211	1.2041	1.1880	1.1729	1.1585	1.1448	1.1318
.900	1.2843	1.2642	1.2455	1.2279	1.2114	1.1958	1.1810	1.1670	1.1536	1.1409
.910	1.2855	1.2656	1.2470	1.2296	1.2132	1.1977	1.1830	1.1691	1.1559	1.1432
.930	1.2884	1.2689	1.2507	1.2335	1.2174	1.2022	1.1877	1.1740	1.1610	1.1485
.950	1.2921	1.2731	1.2553	1.2386	1.2228	1.2079	1.1938	1.1804	1.1676	1.1553
.970	1.2975	1.2792	1.2620	1.2459	1.2306	1.2162	1.2025	1.1894	1.1770	1.1651
.980	1.3014	1.2836	1.2670	1.2512	1.2363	1.2223	1.2089	1.1961	1.1840	1.1723
.990	1.3077	1.2907	1.2748	1.2597	1.2455	1.2320	1.2191	1.2067	1.1950	1.1837
.995	1.3133	1.2971	1.2818	1.2674	1.2537	1.2407	1.2283	1.2164	1.2051	1.1941
.997	1.3171	1.3014	1.2866	1.2726	1.2593	1.2467	1.2346	1.2229	1.2119	1.2012
.999	1.3243	1.3097	1.2957	1.2826	1.2700	1.2581	1.2466	1.2355	1.2251	1.2149

Table 1 (cont'd)

Values of h as a function of gamma (γ) and selected probabilities

P \ γ	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
.001	1.05479	1.04012	1.02614	1.01280	.99996	.98771	.97590	.96460	.95371	.94320
.003	1.05546	1.04093	1.02700	1.01373	1.00101	.98881	.97715	.96588	.95515	.94473
.005	1.05592	1.04145	1.02760	1.01437	1.00171	.98962	.97796	.96685	.95607	.94575
.006	1.05612	1.04163	1.02783	1.01463	1.00200	.98992	.97830	.96713	.95645	.94608
.007	1.05631	1.04190	1.02806	1.01493	1.00227	.99021	.97861	.96750	.95679	.94650
.008	1.05648	1.04210	1.02827	1.01512	1.00252	.99047	.97889	.96780	.95711	.94680
.009	1.05664	1.04225	1.02847	1.01538	1.00275	.99077	.97916	.96808	.95741	.94713
.010	1.05680	1.04245	1.02865	1.01558	1.00297	.99096	.97941	.96835	.95768	.94745
.015	1.05749	1.04320	1.02949	1.01642	1.00393	.99197	.98049	.96950	.95888	.94870
.020	1.05809	1.04385	1.03019	1.01717	1.00474	.99282	.98140	.97045	.95988	.94975
.025	1.05863	1.04438	1.03082	1.01790	1.00546	.99362	.98219	.97123	.96075	.95058
.030	1.05912	1.04495	1.03140	1.01848	1.00611	.99427	.98291	.97200	.96153	.95140
.035	1.05959	1.04543	1.03194	1.01903	1.00671	.99492	.98358	.97276	.96226	.95221
.040	1.06002	1.04588	1.03244	1.01960	1.00728	.99552	.98420	.97335	.96293	.95285
.045	1.06044	1.04635	1.03292	1.02008	1.00781	.99607	.98479	.97401	.96356	.95356
.050	1.06084	1.04681	1.03338	1.02058	1.00832	.99656	.98534	.97457	.96416	.95417
.055	1.06123	1.04720	1.03382	1.02103	1.00881	.99713	.98588	.95508	.96474	.95473
.060	1.06161	1.04758	1.03424	1.02150	1.00928	.99762	.98639	.97566	.96529	.95531
.065	1.06197	1.04806	1.03465	1.02192	1.00974	.99808	.98689	.97615	.96582	.95585
.070	1.06233	1.04841	1.03505	1.02238	1.01018	.99858	.98736	.97663	.96633	.95638
.075	1.06267	1.04873	1.03544	1.02275	1.01061	.99897	.98783	.97710	.96683	.95690
.080	1.06301	1.04910	1.03583	1.02313	1.01103	.99943	.98828	.97760	.96731	.95740
.085	1.06334	1.04946	1.03620	1.02353	1.01144	.99982	.98873	.97806	.96779	.95791
.090	1.06367	1.04986	1.03656	1.02392	1.01184	1.00030	.98916	.97848	.96825	.95833
.095	1.06399	1.05015	1.03692	1.02428	1.01223	1.00068	.98958	.97895	.96870	.95885
.100	1.06431	1.05051	1.03728	1.02468	1.01262	1.00108	.99000	.97935	.96914	.95925
.150	1.06729	1.05366	1.04058	1.02815	1.01621	1.00477	.99385	.98331	.97322	.96346
.200	1.07007	1.05655	1.04363	1.03131	1.01950	1.00817	.99735	.98691	.97691	.96726
.250	1.07277	1.05941	1.04657	1.03441	1.02265	1.01148	1.00069	.99036	.98041	.97081
.300	1.07545	1.06221	1.04948	1.03741	1.02576	1.01470	1.00396	.99370	.98383	.97430
.350	1.07817	1.06501	1.05241	1.04042	1.02887	1.01782	1.00724	.99708	.98725	.97783
.400	1.08096	1.06792	1.05540	1.04345	1.03205	1.02111	1.01057	1.00046	.99072	.98131
.450	1.08386	1.07092	1.05851	1.04666	1.03532	1.02445	1.01400	1.00397	.99429	.98497
.500	1.08690	1.07408	1.06176	1.05007	1.03876	1.02800	1.01759	1.00762	.99801	.98872
.550	1.09014	1.07738	1.06521	1.05357	1.04239	1.03171	1.02138	1.01146	1.00194	.99271
.600	1.09363	1.08098	1.06893	1.05738	1.04629	1.03565	1.02545	1.01557	1.00614	.99697
.631	1.09596	1.08343	1.07140	1.05993	1.04889	1.03830	1.02814	1.01832	1.00893	.99982
.650	1.09746	1.08498	1.07299	1.06158	1.05055	1.04006	1.02988	1.02012	1.01072	1.00162
.700	1.10172	1.08935	1.07750	1.06618	1.05528	1.04486	1.03479	1.02513	1.01580	1.00678
.750	1.10659	1.09433	1.08264	1.07145	1.06066	1.05036	1.04038	1.03085	1.02156	1.01264
.800	1.11231	1.10026	1.08869	1.07765	1.06698	1.05676	1.04693	1.03745	1.02831	1.01945
.850	1.11938	1.10750	1.09614	1.08526	1.07476	1.06472	1.05498	1.04563	1.03661	1.02788
.900	1.12880	1.11723	1.10606	1.09540	1.08511	1.07523	1.06571	1.05648	1.04765	1.03903
.910	1.13116	1.11961	1.10854	1.09791	1.08770	1.07788	1.06838	1.05925	1.05040	1.04185
.930	1.13662	1.12523	1.11430	1.10381	1.09370	1.08398	1.07460	1.06555	1.05681	1.04835
.950	1.14365	1.13245	1.12169	1.11136	1.10141	1.09185	1.08259	1.07365	1.06500	1.05665
.970	1.15368	1.14281	1.13226	1.12216	1.11244	1.10305	1.09401	1.08527	1.07679	1.06857
.980	1.16115	1.15043	1.14014	1.13025	1.12066	1.11143	1.10253	1.09393	1.08557	1.07748
.990	1.17298	1.16261	1.15262	1.14298	1.13370	1.12477	1.11605	1.10766	1.09952	1.09161
.995	1.18374	1.17368	1.16398	1.15465	1.14559	1.13686	1.12840	1.12023	1.11226	1.10458
.997	1.19105	1.18122	1.17171	1.16256	1.15368	1.14511	1.13681	1.12880	1.12096	1.11340
.999	1.20519	1.19580	1.18669	1.17791	1.16940	1.16118	1.15318	1.14543	1.13791	1.13058

Table 1 (cont'd)

Values of h as a function of gamma (γ) and selected probabilities

P \ γ	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
.001	.93315	.92342	.91403	.90497	.89620	.88772	.87951	.87156	.86385	.85637
.003	.93478	.92514	.91584	.90685	.89816	.88977	.88164	.87377	.86613	.85873
.005	.93580	.92621	.91696	.90802	.89938	.89103	.88294	.87511	.86751	.86014
.006	.93622	.92665	.91741	.90850	.89988	.89154	.88347	.87565	.86807	.86071
.007	.93660	.92704	.91783	.90892	.90032	.89200	.88394	.87613	.86857	.86122
.008	.93694	.92741	.91820	.90932	.90072	.89241	.88437	.87657	.86902	.86169
.009	.93727	.92774	.91855	.90968	.90110	.89280	.88477	.87698	.86944	.86212
.010	.93757	.92806	.91888	.91002	.90145	.89316	.88514	.87737	.86983	.86252
.015	.93888	.92942	.92092	.91147	.90295	.89370	.88672	.87899	.87149	.86422
.020	.93966	.93054	.92144	.91266	.90417	.89596	.88801	.88031	.87284	.86559
.025	.94090	.93151	.92244	.91369	.90523	.89705	.88912	.88144	.87400	.86678
.030	.94174	.93238	.92334	.91462	.90618	.89802	.89011	.88246	.87503	.86783
.035	.94251	.93318	.92416	.91546	.90704	.89890	.89102	.88338	.87597	.86879
.040	.94323	.93392	.92493	.91624	.90784	.89972	.89185	.88423	.87684	.86967
.045	.94391	.93461	.92564	.91697	.90859	.90049	.89264	.88503	.87765	.87049
.050	.94455	.93527	.92632	.91767	.90930	.90121	.89337	.88578	.87842	.87127
.055	.94516	.93590	.92696	.91832	.90998	.90190	.89408	.88649	.87914	.87201
.060	.94575	.93650	.92758	.91896	.91062	.90256	.89474	.88718	.87984	.87271
.065	.94631	.93708	.92817	.91956	.91124	.90319	.89539	.88783	.88050	.87339
.070	.94685	.93764	.92874	.92015	.91184	.90379	.89601	.88846	.88114	.87404
.075	.94738	.93818	.92929	.92071	.91241	.90438	.89661	.88907	.88176	.87466
.080	.94789	.93870	.92983	.92126	.91297	.90495	.89719	.88966	.88236	.87527
.085	.94839	.93921	.93035	.92180	.91352	.90551	.89775	.89023	.88294	.87586
.090	.94888	.93971	.93086	.92232	.91405	.90605	.89830	.89079	.88351	.87644
.095	.94936	.94020	.93136	.92282	.91457	.90658	.89884	.89133	.88406	.87700
.100	.94983	.94068	.93185	.92332	.91508	.90709	.89936	.89187	.88460	.87754
.150	.95410	.94505	.93631	.92787	.91970	.91179	.90413	.89671	.88950	.88251
.200	.95796	.94898	.94032	.93194	.92384	.91599	.90839	.90102	.89386	.88692
.250	.96161	.95270	.94409	.93578	.92773	.91993	.91238	.90506	.89795	.89104
.300	.96516	.95632	.94777	.93950	.93151	.92376	.91625	.90897	.90190	.89503
.350	.96871	.95992	.95142	.94321	.93525	.92755	.92008	.91284	.90581	.89897
.400	.97229	.96356	.95511	.94695	.93904	.93138	.92395	.91674	.90974	.90294
.450	.97598	.96729	.95890	.95078	.94292	.93530	.92791	.92074	.91377	.90700
.500	.97981	.97118	.96284	.95476	.94694	.93937	.93201	.92488	.91795	.91121
.550	.98386	.97528	.96699	.95896	.95119	.94365	.93633	.92923	.92234	.91563
.600	.98819	.97967	.97142	.96345	.95571	.94822	.94094	.93388	.92702	.92034
.631	.99106	.98257	.97436	.96641	.95871	.95124	.94399	.93695	.93011	.92346
.650	.99290	.98443	.97624	.96831	.96063	.95318	.94594	.93892	.93209	.92545
.700	.99811	.98971	.98158	.97370	.96607	.95866	.95147	.94449	.93770	.93110
.750	1.00402	.99569	.98762	.97981	.97223	.96487	.95773	.95080	.94405	.93749
.800	1.01095	1.00269	.99470	.98695	.97944	.97214	.96506	.95817	.95147	.94496
.850	1.01946	1.01129	1.00339	.99572	.98828	.98106	.97404	.96722	.96058	.95412
.900	1.03077	1.02273	1.01494	1.00738	1.00004	.99291	.98598	.97924	.97267	.96629
.910	1.03359	1.02559	1.01782	1.01029	1.00298	.99587	.98897	.98224	.97570	.96933
.930	1.04016	1.03222	1.02452	1.01706	1.00980	1.00275	.99589	.98922	.98271	.97639
.950	1.04858	1.04074	1.03313	1.02574	1.01856	1.01158	1.00479	.99818	.99173	.98546
.970	1.06064	1.05294	1.04545	1.03818	1.03111	1.02424	1.01754	1.01102	1.00466	.99847
.980	1.06965	1.06204	1.05465	1.04747	1.04049	1.03370	1.02708	1.02063	1.01432	1.00820
.990	1.08397	1.07653	1.06930	1.06227	1.05543	1.04877	1.04227	1.03594	1.02973	1.02373
.995	1.09707	1.08980	1.08272	1.07583	1.06912	1.06259	1.05621	1.04999	1.04387	1.03800
.997	1.10602	1.09886	1.09189	1.08511	1.07849	1.07205	1.06576	1.05962	1.05355	1.04778
.999	1.12347	1.11654	1.10980	1.10322	1.09680	1.09055	1.08443	1.07846	1.07239	1.06691

Table 1 (cont'd)

Values of h as a function of gamma (γ) and selected probabilities

P \ γ	4.0	4.1	4.2	4.3	4.4
.001	.84912	.84207	.83522	.82856	.82208
.003	.85154	.84456	.84777	.83118	.82476
.005	.85299	.84605	.83930	.83274	.82635
.006	.85358	.84664	.83991	.83336	.82698
.007	.85410	.84718	.84045	.83391	.82755
.008	.85458	.84767	.84095	.83442	.82807
.009	.85502	.84811	.84141	.83489	.82854
.010	.85542	.84853	.84183	.83532	.82898
.015	.85716	.85030	.84363	.83715	.83084
.020	.85856	.85173	.84509	.83863	.83234
.025	.85977	.85295	.84633	.83989	.83363
.030	.86084	.85404	.84744	.84101	.83476
.035	.86181	.85503	.84844	.84203	.83579
.040	.86271	.85594	.84936	.84297	.83674
.045	.86354	.85679	.85023	.84384	.83763
.050	.86433	.85759	.85104	.84466	.83846
.055	.86508	.85835	.85181	.84544	.83924
.060	.86580	.85907	.85254	.84618	.83999
.065	.86648	.85977	.85324	.84689	.84071
.070	.86714	.86043	.85392	.84758	.84140
.075	.86771	.86108	.85457	.84823	.84207
.080	.86839	.86170	.85520	.84887	.84271
.085	.86899	.96231	.85581	.84949	.84334
.090	.86957	.86290	.85641	.85009	.84395
.095	.87014	.86347	.85699	.85068	.84454
.100	.97069	.86403	.85755	.85125	.84512
.150	.87571	.86911	.86268	.85643	.85035
.200	.88016	.87361	.86722	.86101	.85496
.250	.88433	.87781	.87146	.86528	.85926
.300	.88835	.88186	.87555	.86940	.86340
.350	.89233	.88587	.87958	.87346	.86749
.400	.89633	.88990	.88364	.87754	.87160
.450	.90042	.89402	.88778	.88171	.87579
.500	.90466	.89828	.89207	.88603	.88013
.550	.90911	.90276	.89658	.89055	.88468
.600	.91385	.90753	.90138	.89538	.88953
.631	.91699	.91068	.90455	.89856	.89273
.650	.91899	.91270	.90658	.90060	.89478
.700	.92467	.91841	.91231	.90637	.90057
.750	.93110	.92487	.91881	.91289	.90712
.800	.93861	.93242	.92639	.92051	.91478
.850	.94782	.94168	.93570	.92986	.92416
.900	.96005	.95398	.94806	.94228	.93663
.910	.96311	.95706	.95115	.94537	.93974
.930	.97022	.96419	.95832	.95258	.94698
.950	.97934	.97337	.96754	.96184	.95628
.970	.99243	.98653	.98076	.97513	.96963
.980	1.00222	.99637	.99066	.98508	.97962
.990	1.01784	1.01209	1.00647	1.00097	.99559
.995	1.03220	1.02654	1.02101	1.01559	1.01029
.997	1.04205	1.03646	1.03098	1.02562	1.02038
.999	1.06133	1.05586	1.05050	1.04526	1.04012

Table 2

Quantiles for selected probabilities and shape parameter values

P \ γ	.01	.02	.03	.04	.05
.01			.12394072E-66	.57987583E-50	.58446321E-40
.05		.50686677E-65	.24672558E-43	.17281647E-32	.55738755E-26
.10		.57068125E-50	.26702220E-33	.57987583E-25	.58424026E-20
.15	.23014364E-82	.36387863E-41	.19781353E-27	.14621980E-20	.19434868E-16
.20	.71758381E-70	.64252996E-35	.28898849E-23	.19457400E-17	.61285408E-14
.25	.35226860E-60	.45018812E-30	.49105858E-20	.51503319E-15	.53156618E-12
.30	.29174172E-52	.40969092E-26	.21407932E-17	.49132217E-13	.20378972E-10
.35	.14443668E-45	.91158216E-23	.36487897E-15	.23175873E-11	.44475215E-09
.40	.90964555E-40	.72327819E-20	.31276180E-13	.65288212E-10	.64262409E-08
.45	.11861086E-34	.26121932E-17	.15859802E-11	.12406459E-08	.67765317E-07
.50	.44655350E-30	.50686667E-15	.53155408E-10	.17281646E-07	.55738784E-06
.55	.61537807E-26	.59501510E-13	.12743937E-08	.18724158E-06	.37498381E-05
.60	.36982657E-22	.46127086E-11	.23169803E-07	.16486063E-05	.21369336E-04
.65	.11071557E-18	.25238371E-09	.33393304E-06	.12194781E-04	.10594199E-03
.70	.18309525E-15	.10263500E-07	.39489687E-05	.77771144E-04	.46656369E-03
.75	.18155132E-12	.32318925E-06	.39379731E-04	.43656497E-03	.18567357E-02
.80	.11531127E-09	.81451000E-05	.33860213E-03	.21953358E-02	.67819976E-02
.85	.49518290E-07	.16881528E-03	.25601005E-02	.10069388E-01	.23155919E-01
.90	.15035712E-04	.29496744E-02	.17456650E-01	.43384135E-01	.76317114E-01
.95	.33514566E-02	.45909824E-01	.11613830E 00	.19189907E 00	.26593230E 00
.99	.20720132E 00	.55888579E 00	.77456396E 00	.94529479E 00	.10876274E 01

P \ γ	.06	.07	.08	.09	.10
.01	.27341005E-33	.15924982E-28	.59818684E-25	.36133847E-22	.60730484E-20
.05	.12198728E-21	.15394469E-18	.32655833E-16	.21098305E-14	.59307113E-13
.10	.12690570E-16	.30746325E-14	.18916329E-12	.46668657E-11	.60730484E-10
.15	.10922837E-13	.10078192E-11	.30059212E-10	.42227449E-09	.35020257E-08
.20	.13202243E-11	.61407545E-10	.10957508E-08	.10322931E-07	.62188019E-07
.25	.54427028E-10	.14881630E-08	.17827357E-07	.12319157E-06	.57917133E-06
.30	.11363237E-08	.20128488E-07	.17412158E-06	.93405608E-06	.35860860E-05
.35	.14834807E-07	.18205136E-06	.11958808E-05	.51787848E-05	.16753047E-04
.40	.13734548E-06	.12264525E-05	.63473040E-05	.22834414E-04	.63684214E-04
.45	.97804035E-06	.65978622E-05	.27669697E-04	.84523175E-04	.20683000E-03
.50	.56621782E-05	.29722892E-04	.10327696E-03	.27256324E-03	.59339110E-03
.55	.27724943E-04	.11599791E-03	.34002214E-03	.78630430E-03	.15404287E-02
.60	.11822711E-03	.40216371E-03	.10095629E-02	.20700208E-02	.36844507E-02
.65	.44897382E-03	.12628553E-02	.27501669E-02	.50513369E-02	.82373349E-02
.70	.15455444E-02	.36483960E-02	.69720686E-02	.11577229E-01	.17427776E-01
.75	.48959754E-02	.98321906E-02	.16663985E-01	.25230481E-01	.35306358E-01
.80	.14484238E-01	.25072516E-01	.38075347E-01	.52994412E-01	.69389883E-01
.85	.40770497E-01	.61642753E-01	.84724849E-01	.10924621E 00	.13466330E 00
.90	.11287551E 00	.15104388E 00	.18968585E 00	.22817082E 00	.26615455E 00
.95	.33627848E 00	.40263162E 00	.46520323E 00	.52434795E 00	.58043511E 00
.99	.12104562E 01	.13190668E 01	.14168493E 01	.15061075E 01	.15884778E 01

Table 2 (cont'd)

Quantiles for selected probabilities and shape parameter values

P \ γ	.15	.20	.25	.30	.35
.01	.29241836E-13	.65254808E-10	.67496979E-08	.15022227E-06	.13890538E-05
.05	.13359945E-08	.20392131E-06	.42185754E-05	.32110347E-04	.13798128E-03
.10	.13572860E-06	.65255163E-05	.67500624E-04	.32372462E-03	.10004220E-02
.15	.20258804E-05	.49554917E-04	.34179690E-03	.12515739E-02	.31915788E-02
.20	.13789330E-04	.20885173E-03	.10808857E-02	.32703395E-02	.72826141E-02
.25	.61041859E-04	.63759263E-03	.26421771E-02	.68998026E-02	.13844491E-01
.30	.20585310E-03	.15877907E-02	.54913025E-02	.12726658E-01	.23474091E-01
.35	.57545172E-03	.34371293E-02	.10211699E-01	.21416918E-01	.36822988E-01
.40	.14026009E-02	.67195668E-02	.17522387E-01	.33739793E-01	.54633354E-01
.45	.30802355E-02	.12163741E-01	.28308739E-01	.50606536E-01	.77786140E-01
.50	.62348058E-02	.20746339E-01	.43673802E-01	.73131136E-01	.10736949E 00
.55	.11827103E-01	.33773109E-01	.65023053E-01	.10272545E 00	.14478095E 00
.60	.21298804E-01	.53010603E-01	.94205874E-01	.14125250E 00	.19188839E 00
.65	.36803540E-01	.80913131E-01	.13375913E 00	.19128463E 00	.25129798E 00
.70	.61608021E-01	.12103759E 00	.18734823E 00	.25656591E 00	.32683073E 00
.75	.10086478E 00	.17885916E 00	.26062600E 00	.34289946E 00	.42444158E 00
.80	.16328679E 00	.26354363E 00	.36308525E 00	.46007389E 00	.55418999E 00
.85	.26537126E 00	.39239766E 00	.51268651E 00	.62662755E 00	.73514138E 00
.90	.44479853E 00	.60490232E 00	.75039287E 00	.88481077E 00	.10106953E 01
.95	.82558744E 00	.10305258E 01	.12101161E 01	.13723499E 01	.15219584E 01
.99	.19305934E 01	.22023049E 01	.24338854E 01	.26394092E 01	.28265841E 01

P \ γ	.40	.45	.50	.55	.60
.01	.74153940E-05	.27439797E-04	.78543929E-04	.18649439E-03	.38483493E-03
.05	.41465372E-03	.98159722E-03	.19660700E-02	.34869415E-02	.56448343E-02
.10	.23488772E-02	.45916553E-02	.78953870E-02	.12366732E-01	.18060438E-01
.15	.64919025E-02	.11358089E-01	.17882890E-01	.26076151E-01	.35894098E-01
.20	.13392235E-01	.21678219E-01	.32092377E-01	.44516986E-01	.58803353E-01
.25	.23564946E-01	.35943238E-01	.50765522E-01	.67788174E-01	.86771720E-01
.30	.37541886E-01	.54586943E-01	.74235931E-01	.96139061E-01	.11998882E 00
.35	.55911714E-01	.78122844E-01	.10295006E 00	.12996633E 00	.15882163E 00
.40	.79361887E-01	.10718363E 00	.13749795E 00	.16983328E 00	.20382258E 00
.45	.10873061E 00	.14257143E 00	.17865858E 00	.21650947E 00	.25576405E 00
.50	.14507814E 00	.18532838E 00	.22746821E 00	.27103638E 00	.31570202E 00
.55	.18979205E 00	.23684263E 00	.28532593E 00	.33483270E 00	.38508138E 00
.60	.24475234E 00	.29901585E 00	.35416315E 00	.40986558E 00	.46590925E 00
.65	.31260631E 00	.37454349E 00	.43672857E 00	.49894020E 00	.56104913E 00
.70	.39725716E 00	.46741396E 00	.53709709E 00	.60621791E 00	.67474797E 00
.75	.50480611E 00	.58387147E 00	.66165185E 00	.73821644E 00	.81365358E 00
.80	.64557102E 00	.73447837E 00	.82118721E 00	.90595089E 00	.98899180E 00
.85	.83910173E 00	.93923492E 00	.10361254E 01	.11302414E 01	.12219601E 01
.90	.11298428E 01	.12435490E 01	.13527717E 01	.14582362E 01	.15605034E 01
.95	.16619620E 01	.17944039E 01	.19207294E 01	.20419986E 01	.21590121E 01
.99	.30000967E 01	.31630125E 01	.33174483E 01	.34649288E 01	.36065893E 01

Table 2 (cont'd)

Quantiles for selected probabilities and shape parameter values

P \ γ	.65	.70	.75	.80	.85
.01	.71277933E-03	.12126230E-02	.19272509E-02	.28980766E-02	.41635370E-02
.05	.85181609E-02	.12163199E-01	.16616388E-01	.21897533E-01	.28013140E-01
.10	.24991261E-01	.33145498E-01	.42490486E-01	.52981822E-01	.64568542E-01
.15	.47261915E-01	.60089280E-01	.74280323E-01	.89739657E-01	.10637579E 00
.20	.74794705E-01	.92338598E-01	.11129296E 00	.13152850E 00	.15292912E 00
.25	.10749592E 00	.12976492E 00	.15340753E 00	.17827535E 00	.20424020E 00
.30	.14552276E 00	.17251980E 00	.20079461E 00	.23019178E 00	.26058067E 00
.35	.18923209E 00	.22096777E 00	.25384184E 00	.28770163E 00	.32242151E 00
.40	.23917969E 00	.27568007E 00	.31314598E 00	.35143535E 00	.39043351E 00
.45	.29615107E 00	.33746366E 00	.37954205E 00	.42226125E 00	.46552234E 00
.50	.36122400E 00	.40742375E 00	.45416698E 00	.50135123E 00	.54889726E 00
.55	.43587450E 00	.48707106E 00	.53856877E 00	.59029232E 00	.64218556E 00
.60	.52215250E 00	.57850019E 00	.63488778E 00	.69127126E 00	.74762055E 00
.65	.62298069E 00	.68469319E 00	.74616519E 00	.80738779E 00	.86835984E 00
.70	.74269022E 00	.81006375E 00	.87689542E 00	.94321522E 00	.10090538E 01
.75	.88805467E 00	.96150709E 00	.10340914E 01	.11058806E 01	.11769400E 01
.80	.10705016E 01	.11506443E 01	.12295606E 01	.13073710E 01	.13841796E 01
.85	.13115878E 01	.13993760E 01	.14855333E 01	.15702343E 01	.16536264E 01
.90	.16600159E 01	.17571285E 01	.18521300E 01	.19452584E 01	.20367126E 01
.95	.22723894E 01	.23826200E 01	.24900976E 01	.25951436E 01	.26980239E 01
.99	.37432990E 01	.38757387E 01	.40044516E 01	.41298791E 01	.42523845E 01

P \ γ	.90	.95	1.0	1.1	1.2
.01	.57581294E-02	.77119084E-02	.10050336E-01	.15960789E-01	.23608723E-01
.05	.34959469E-01	.42725146E-01	.51293294E-01	.70751765E-01	.93145199E-01
.10	.77196721E-01	.90811875E-01	.10536052E 00	.13705463E 00	.17189837E 00
.15	.12410289E 00	.14284155E 00	.16251893E 00	.20442986E 00	.24937278E 00
.20	.17539136E 00	.19882333E 00	.22314355E 00	.27416759E 00	.32797593E 00
.25	.23119136E 00	.25903302E 00	.28768207E 00	.34712201E 00	.40903367E 00
.30	.29185095E 00	.32390901E 00	.35667494E 00	.42406562E 00	.49357816E 00
.35	.35789736E 00	.39404222E 00	.43078292E 00	.50581289E 00	.58259093E 00
.40	.43004703E 00	.47019900E 00	.51082562E 00	.59329750E 00	.67712632E 00
.45	.50924612E 00	.55336848E 00	.59783700E 00	.68764683E 00	.77840825E 00
.50	.59674305E 00	.64483946E 00	.69314718E 00	.79027528E 00	.88793621E 00
.55	.69420613E 00	.74632178E 00	.79850770E 00	.90301779E 00	.10076279E 00
.60	.80391524E 00	.86014162E 00	.91629073E 00	.10283371E 01	.11400344E 01
.65	.92909497E 00	.98956968E 00	.10498221E 00	.11696666E 01	.12886927E 01
.70	.10744409E 01	.11394051E 01	.12039728E 00	.13320160E 01	.14587463E 01
.75	.12473273E 01	.13170975E 01	.13862943E 01	.15231332E 01	.16581295E 01
.80	.14600763E 01	.15351395E 01	.16094379E 01	.17559741E 01	.19000888E 01
.85	.17358353E 01	.18169687E 01	.18971200E 01	.20547908E 01	.22093856E 01
.90	.21266601E 01	.22152434E 01	.23025851E 01	.24739531E 01	.26414604E 01
.95	.27989611E 01	.28981436E 01	.29957323E 01	.31866641E 01	.33725633E 01
.99	.43722707E 01	.44897934E 01	.46051702E 01	.48302094E 01	.50486095E 01

Table 2 (cont'd)

Quantiles for selected probabilities and shape parameter values

P \ γ	1.3	1.4	1.5
.01	.33058070E-01	.44330292E-01	.57415901E-01
.05	.11827039E 00	.14592651E 00	.17592316E 00
.10	.20955619E 00	.24973648E 00	.29218719E 00
.15	.29696530E 00	.34689081E 00	.39888572E 00
.20	.38418046E 00	.44246888E 00	.50258701E 00
.25	.47304661E 00	.53886879E 00	.60626645E 00
.30	.56487458E 00	.63769285E 00	.71182612E 00
.35	.66082148E 00	.74027900E 00	.82078780E 00
.40	.76206620E 00	.84793267E 00	.93458420E 00
.45	.86993056E 00	.96207392E 00	.10547333E 01
.50	.98599905E 00	.10843715E 01	.11829869E 01
.55	.11122708E 01	.12169045E 01	.13215026E 01
.60	.12513831E 01	.13623940E 01	.14730830E 01
.65	.14069732E 01	.15245743E 01	.16415562E 01
.70	.15843122E 01	.17088396E 01	.18324354E 01
.75	.17915140E 01	.19234761E 01	.20541725E 01
.80	.20421025E 01	.21822736E 01	.23208138E 01
.85	.23613257E 01	.25109475E 01	.26585239E 01
.90	.28056478E 01	.29669437E 01	.31256943E 01
.95	.35544295E 01	.37325150E 01	.39073640E 01
.99	.52613106E 01	.54690535E 01	.56724334E 01

Table 3

Probabilities for selected quantiles and shape parameter values

$\tau \diagdown \gamma$	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10E-25	.5527	.3054	.1687	.0932	.0515	.0284	.0157	.0087	.0048
.10E-24	.5656	.3198	.1808	.1022	.0578	.0326	.0184	.0104	.0059
.10E-23	.5787	.3349	.1937	.1121	.0648	.0375	.0217	.0125	.0072
.10E-22	.5922	.3506	.2076	.1229	.0727	.0430	.0255	.0151	.0089
.10E-21	.6060	.3672	.2224	.1347	.0816	.0494	.0299	.0181	.0110
.10E-20	.6201	.3845	.2383	.1477	.0916	.0567	.0351	.0218	.0135
.10E-19	.6346	.4026	.2554	.1620	.1027	.0651	.0413	.0262	.0166
.10E-18	.6493	.4216	.2737	.1776	.1153	.0748	.0485	.0315	.0204
.10E-17	.6645	.4414	.2932	.1947	.1293	.0859	.0570	.0378	.0251
.10E-16	.6799	.4622	.3142	.2135	.1451	.0986	.0670	.0455	.0309
.10E-15	.6958	.4840	.3367	.2341	.1628	.1132	.0787	.0547	.0380
.10E-14	.7120	.5068	.3607	.2567	.1827	.1300	.0924	.0657	.0468
.10E-13	.7286	.5307	.3865	.2815	.2050	.1492	.1086	.0790	.0575
.10E-12	.7455	.5557	.4142	.3087	.2300	.1713	.1276	.0950	.0708
.10E-11	.7629	.5819	.4438	.3384	.2580	.1967	.1499	.1142	.0871
.10E-10	.7807	.6094	.4756	.3711	.2895	.2258	.1761	.1374	.1071
.10E-09	.7989	.6381	.5096	.4069	.3248	.2593	.2069	.1651	.1318
.10E-08	.8175	.6681	.5460	.4461	.3645	.2977	.2431	.1985	.1621
.10E-07	.8365	.6996	.5851	.4892	.4089	.3418	.2857	.2387	.1994
.10E-06	.8560	.7326	.6269	.5364	.4588	.3925	.3356	.2870	.2454
.10E-05	.8759	.7671	.6717	.5881	.5148	.4506	.3943	.3450	.3018
.10E-04	.8963	.8033	.7198	.6449	.5776	.5174	.4633	.4148	.3714
.10E-03	.9172	.8411	.7713	.7071	.6481	.5940	.5443	.4987	.4569
.10E-02	.9386	.8808	.8264	.7753	.7272	.6820	.6395	.5995	.5620
.10E-01	.9603	.9221	.8853	.8498	.8156	.7826	.7509	.7203	.6909
.10E 00	.9819	.9639	.9462	.9286	.9113	.8941	.8772	.8604	.8439
.12E 00	.9835	.9671	.9508	.9347	.9188	.9030	.8873	.8718	.8565
.14E 00	.9848	.9697	.9547	.9398	.9250	.9104	.8958	.8814	.8671
.16E 00	.9859	.9719	.9580	.9442	.9304	.9167	.9031	.8896	.8762
.18E 00	.9869	.9739	.9609	.9480	.9351	.9223	.9095	.8968	.8842
.20E 00	.9878	.9756	.9634	.9513	.9392	.9272	.9151	.9032	.8913
.22E 00	.9885	.9771	.9657	.9543	.9429	.9315	.9202	.9089	.8976
.24E 00	.9892	.9785	.9677	.9569	.9462	.9354	.9247	.9140	.9033
.26E 00	.9899	.9797	.9695	.9593	.9492	.9390	.9288	.9186	.9085
.28E 00	.9904	.9808	.9712	.9615	.9519	.9422	.9326	.9229	.9132
.30E 00	.9909	.9818	.9727	.9635	.9544	.9452	.9360	.9268	.9176
.32E 00	.9914	.9828	.9741	.9654	.9567	.9479	.9392	.9304	.9216
.34E 00	.9918	.9836	.9754	.9671	.9588	.9505	.9421	.9337	.9253
.36E 00	.9922	.9844	.9766	.9687	.9608	.9528	.9448	.9368	.9288
.38E 00	.9926	.9852	.9777	.9701	.9626	.9550	.9473	.9397	.9320
.40E 00	.9930	.9859	.9787	.9715	.9643	.9570	.9497	.9424	.9350
.42E 00	.9933	.9865	.9797	.9728	.9659	.9589	.9519	.9449	.9378
.44E 00	.9936	.9871	.9806	.9740	.9674	.9607	.9540	.9472	.9404
.46E 00	.9939	.9877	.9814	.9751	.9688	.9624	.9559	.9494	.9429
.48E 00	.9941	.9882	.9822	.9762	.9701	.9639	.9578	.9515	.9453
.50E 00	.9944	.9887	.9830	.9772	.9713	.9654	.9595	.9535	.9475
.52E 00	.9946	.9892	.9837	.9781	.9725	.9668	.9611	.9554	.9496
.54E 00	.9948	.9896	.9843	.9790	.9736	.9682	.9627	.9571	.9515
.56E 00	.9950	.9900	.9849	.9798	.9746	.9694	.9641	.9588	.9534
.58E 00		.9904	.9855	.9806	.9756	.9706	.9655	.9604	.9552

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Table 3 (cont'd)

Probabilities for selected quantiles and shape parameter values

$\tau \backslash \gamma$	.01	.02	.03	.04	.05	.06	.07	.08	.09
.60E 00		.9908	.9861	.9814	.9766	.9717	.9668	.9619	.9569
.62E 00		.9911	.9866	.9821	.9775	.9728	.9681	.9633	.9585
.64E 00		.9915	.9872	.9828	.9783	.9738	.9693	.9647	.9600
.66E 00		.9918	.9876	.9834	.9791	.9748	.9704	.9660	.9615
.68E 00		.9921	.9881	.9840	.9799	.9757	.9715	.9672	.9629
.70E 00		.9924	.9885	.9846	.9806	.9766	.9725	.9684	.9642
.72E 00		.9927	.9890	.9852	.9813	.9774	.9735	.9695	.9654
.74E 00		.9930	.9894	.9857	.9820	.9782	.9744	.9706	.9667
.76E 00		.9932	.9897	.9862	.9826	.9790	.9753	.9716	.9678
.78E 00		.9935	.9901	.9867	.9832	.9797	.9762	.9726	.9689
.80E 00		.9937	.9904	.9872	.9838	.9804	.9770	.9735	.9700
.82E 00		.9939	.9908	.9876	.9844	.9811	.9778	.9744	.9710
.84E 00		.9941	.9911	.9880	.9849	.9817	.9785	.9753	.9720
.86E 00		.9943	.9914	.9884	.9854	.9824	.9793	.9761	.9729
.88E 00		.9945	.9917	.9888	.9859	.9830	.9799	.9769	.9738
.90E 00		.9947	.9920	.9892	.9864	.9835	.9806	.9777	.9747
.92E 00		.9949	.9922	.9896	.9868	.9841	.9812	.9784	.9755
.94E 00		.9950	.9925	.9899	.9873	.9846	.9819	.9791	.9763
.96E 00			.9928	.9902	.9877	.9851	.9825	.9798	.9770
.98E 00			.9930	.9906	.9881	.9856	.9830	.9804	.9778
.10E 01			.9932	.9909	.9885	.9860	.9836	.9810	.9785
.11E 01			.9942	.9922	.9902	.9881	.9860	.9838	.9816
.12E 01			.9951	.9934	.9916	.9898	.9880	.9862	.9843
.13E 01				.9943	.9928	.9913	.9897	.9881	.9865
.14E 01				.9951	.9938	.9925	.9911	.9897	.9883
.15E 01				.9958	.9947	.9935	.9923	.9911	.9899
.16E 01					.9954	.9944	.9934	.9923	.9913
.17E 01						.9951	.9942	.9933	.9924
.18E 01							.9950	.9942	.9934
.19E 01							.9956	.9949	.9942
.20E 01								.9956	.9950
.21E 01									.9956

$\tau \backslash \gamma$	.10	.15	.20	.25	.30	.35	.40	.45	.50
.10E-25	.0026	.0001							
.10E-24	.0033	.0002							
.10E-23	.0042	.0003							
.10E-22	.0053	.0004							
.10E-21	.0066	.0005							
.10E-20	.0083	.0008							
.10E-19	.0105	.0011	.0001						
.10E-18	.0132	.0015	.0002						
.10E-17	.0167	.0021	.0003						
.10E-16	.0210	.0030	.0004						
.10E-15	.0264	.0043	.0007	.0001					
.10E-14	.0332	.0060	.0011	.0002					
.10E-13	.0418	.0085	.0017	.0003					

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Table 3 (cont'd)

Probabilities for selected quantiles and shape parameter values

$\tau \backslash \gamma$	.10	.15	.20	.25	.30	.35	.40	.45	.50
.10E-12	.0527	.0120	.0027	.0006	.0001				
.10E-11	.0663	.0170	.0043	.0011	.0003	.0001			
.10E-10	.0835	.0240	.0069	.0020	.0006	.0002			
.10E-09	.1051	.0339	.0109	.0035	.0011	.0004	.0001		
.10E-08	.1323	.0479	.0173	.0062	.0022	.0008	.0003	.0001	
.10E-07	.1666	.0676	.0274	.0110	.0044	.0018	.0007	.0003	.0001
.10E-06	.2097	.0955	.0434	.0196	.0089	.0040	.0018	.0008	.0004
.10E-05	.2640	.1349	.0687	.0349	.0177	.0089	.0045	.0023	.0011
.10E-04	.3324	.1906	.1089	.0620	.0352	.0200	.0113	.0063	.0036
.10E-03	.4185	.2692	.1726	.1103	.0703	.0448	.0283	.0179	.0113
.10E-02	.5268	.3802	.2735	.1962	.1402	.1000	.0711	.0504	.0357
.10E-01	.6626	.5365	.4329	.3482	.2792	.2233	.1781	.1417	.1125
.10E 00	.8276	.7491	.6760	.6083	.5459	.4886	.4362	.3885	.3453
.12E 00	.8413	.7680	.6989	.6343	.5740	.5182	.4667	.4192	.3758
.14E 00	.8529	.7840	.7185	.6567	.5986	.5442	.4936	.4467	.4033
.16E 00	.8629	.7979	.7356	.6764	.6203	.5674	.5179	.4715	.4284
.18E 00	.8717	.8101	.7508	.6940	.6398	.5884	.5399	.4943	.4515
.20E 00	.8794	.8210	.7644	.7099	.6575	.6076	.5601	.5152	.4729
.22E 00	.8864	.8309	.7768	.7243	.6737	.6251	.5788	.5347	.4929
.24E 00	.8926	.8398	.7880	.7375	.6885	.6414	.5961	.5528	.5116
.26E 00	.8983	.8479	.7983	.7496	.7023	.6564	.6122	.5698	.5292
.28E 00	.9036	.8554	.8077	.7609	.7151	.6705	.6273	.5857	.5457
.30E 00	.9084	.8623	.8165	.7713	.7270	.6836	.6415	.6007	.5614
.32E 00	.9128	.8687	.8247	.7811	.7381	.6960	.6548	.6149	.5763
.34E 00	.9169	.8746	.8323	.7902	.7485	.7076	.6675	.6284	.5904
.36E 00	.9207	.8801	.8394	.7987	.7584	.7185	.6794	.6411	.6039
.38E 00	.9243	.8853	.8461	.8068	.7676	.7289	.6907	.6532	.6167
.40E 00	.9276	.8902	.8523	.8143	.7764	.7387	.7014	.6648	.6289
.42E 00	.9307	.8947	.8582	.8215	.7847	.7480	.7117	.6758	.6406
.44E 00	.9336	.8990	.8638	.8282	.7925	.7568	.7214	.6863	.6518
.46E 00	.9364	.9031	.8691	.8346	.8000	.7653	.7307	.6964	.6625
.48E 00	.9390	.9069	.8741	.8407	.8071	.7733	.7395	.7060	.6728
.50E 00	.9414	.9105	.8788	.8465	.8138	.7809	.7480	.7152	.6827
.52E 00	.9437	.9139	.8833	.8520	.8202	.7882	.7561	.7241	.6922
.54E 00	.9459	.9172	.8875	.8572	.8264	.7952	.7639	.7325	.7013
.56E 00	.9480	.9202	.8916	.8622	.8322	.8019	.7713	.7407	.7101
.58E 00	.9500	.9232	.8954	.8669	.8378	.8083	.7785	.7485	.7185
.60E 00	.9518	.9260	.8991	.8715	.8432	.8145	.7854	.7560	.7267
.62E 00	.9536	.9286	.9026	.8758	.8483	.8203	.7919	.7633	.7345
.64E 00	.9553	.9312	.9060	.8800	.8533	.8260	.7983	.7703	.7421
.66E 00	.9569	.9336	.9092	.8840	.8580	.8314	.8044	.7770	.7494
.68E 00	.9585	.9359	.9123	.8878	.8625	.8366	.8102	.7835	.7565
.70E 00	.9599	.9381	.9152	.8914	.8669	.8416	.8159	.7897	.7633
.72E 00	.9614	.9402	.9180	.8949	.8710	.8465	.8213	.7958	.7699
.74E 00	.9627	.9422	.9207	.8983	.8751	.8511	.8266	.8016	.7762
.76E 00	.9640	.9442	.9233	.9015	.8789	.8556	.8316	.8072	.7824
.78E 00	.9652	.9460	.9258	.9046	.8826	.8599	.8365	.8126	.7883
.80E 00	.9664	.9478	.9282	.9076	.8862	.8641	.8412	.8179	.7941
.82E 00	.9675	.9495	.9305	.9105	.8897	.8681	.8458	.8230	.7997

Table 3 (cont'd)

Probabilities for selected quantiles and shape parameter values

$\tau \diagdown \gamma$	.10	.15	.20	.25	.30	.35	.40	.45	.50
.84E 00	.9686	.9512	.9327	.9133	.8930	.8719	.8502	.8279	.8051
.86E 00	.9696	.9528	.9348	.9159	.8962	.8757	.8544	.8326	.8103
.88E 00	.9706	.9543	.9369	.9185	.8993	.8793	.8585	.8372	.8154
.90E 00	.9716	.9557	.9388	.9210	.9023	.8827	.8625	.8417	.8203
.92E 00	.9725	.9571	.9407	.9234	.9051	.8861	.8664	.8460	.8250
.94E 00	.9734	.9585	.9425	.9257	.9079	.8894	.8701	.8502	.8297
.96E 00	.9743	.9598	.9443	.9279	.9106	.8925	.8737	.8542	.8341
.98E 00	.9751	.9610	.9460	.9300	.9132	.8955	.8772	.8581	.8385
.10E 01	.9759	.9622	.9476	.9321	.9157	.8985	.8805	.8619	.8427
.11E 01	.9794	.9676	.9550	.9414	.9270	.9118	.8959	.8792	.8620
.12E 01	.9823	.9722	.9611	.9493	.9366	.9232	.9090	.8942	.8787
.13E 01	.9848	.9760	.9664	.9560	.9448	.9329	.9203	.9070	.8931
.14E 01	.9869	.9792	.9708	.9617	.9518	.9413	.9301	.9182	.9057
.15E 01	.9887	.9820	.9746	.9666	.9579	.9485	.9386	.9279	.9167
.16E 01	.9902	.9843	.9779	.9708	.9631	.9548	.9459	.9364	.9264
.17E 01	.9914	.9863	.9807	.9744	.9676	.9603	.9523	.9438	.9348
.18E 01	.9926	.9881	.9831	.9776	.9716	.9650	.9579	.9503	.9422
.19E 01	.9935	.9896	.9852	.9803	.9750	.9692	.9628	.9560	.9487
.20E 01	.9943	.9909	.9870	.9827	.9780	.9728	.9671	.9611	.9545
.21E 01	.9950	.9920	.9886	.9848	.9806	.9760	.9709	.9655	.9596
.22E 01		.9930	.9900	.9866	.9829	.9787	.9742	.9694	.9641
.23E 01		.9938	.9912	.9882	.9849	.9812	.9772	.9728	.9680
.24E 01		.9946	.9922	.9896	.9866	.9833	.9797	.9758	.9715
.25E 01		.9952	.9931	.9908	.9882	.9852	.9820	.9785	.9747
.26E 01			.9939	.9919	.9895	.9869	.9840	.9809	.9774
.27E 01			.9947	.9928	.9907	.9884	.9858	.9830	.9799
.28E 01			.9953	.9936	.9918	.9897	.9874	.9848	.9820
.29E 01				.9943	.9927	.9908	.9888	.9865	.9840
.30E 01				.9950	.9935	.9918	.9900	.9879	.9857
.35E 01					.9964	.9954	.9944	.9932	.9918
.40E 01						.9974	.9968	.9961	.9953

$\tau \diagdown \gamma$	.55	.60	.65	.70	.75	.80	.85	.90	.95
.10E-06	.0002	.0001							
.10E-05	.0006	.0003	.0001						
.10E-04	.0020	.0011	.0006	.0003	.0002	.0001			
.10E-03	.0071	.0045	.0028	.0017	.0011	.0007	.0004	.0003	.0002
.10E-02	.0252	.0177	.0125	.0087	.0061	.0043	.0030	.0021	.0014
.10E-01	.0890	.0704	.0555	.0436	.0343	.0268	.0210	.0164	.0128
.10E 00	.3062	.2709	.2392	.2108	.1855	.1628	.1427	.1249	.1091
.12E 00	.3361	.3000	.2673	.2376	.2109	.1868	.1652	.1458	.1285
.14E 00	.3634	.3267	.2932	.2626	.2347	.2095	.1866	.1660	.1474
.16E 00	.3884	.3514	.3173	.2860	.2573	.2311	.2072	.1854	.1657
.18E 00	.4116	.3744	.3400	.3081	.2788	.2517	.2270	.2043	.1836
.20E 00	.4332	.3960	.3614	.3291	.2992	.2716	.2460	.2226	.2010
.22E 00	.4534	.4164	.3816	.3491	.3188	.2906	.2645	.2403	.2180
.24E 00	.4725	.4356	.4008	.3681	.3375	.3089	.2823	.2575	.2346

Table 3 (cont'd)

Probabilities for selected quantiles and shape parameter values

τ \ γ	.55	.60	.65	.70	.75	.80	.85	.90	.95
.26E 00	.4905	.4538	.4190	.3863	.3555	.3266	.2995	.2743	.2508
.28E 00	.5075	.4711	.4365	.4037	.3727	.3436	.3162	.2906	.2666
.30E 00	.5237	.4876	.4532	.4204	.3894	.3601	.3324	.3064	.2820
.32E 00	.5391	.5033	.4691	.4365	.4054	.3760	.3481	.3218	.2971
.34E 00	.5537	.5184	.4844	.4519	.4209	.3914	.3634	.3369	.3118
.36E 00	.5677	.5328	.4991	.4668	.4359	.4063	.3782	.3515	.3262
.38E 00	.5811	.5466	.5132	.4811	.4503	.4208	.3926	.3658	.3403
.40E 00	.5939	.5598	.5268	.4950	.4643	.4349	.4067	.3797	.3541
.42E 00	.6062	.5726	.5399	.5083	.4778	.4485	.4203	.3933	.3675
.44E 00	.6179	.5848	.5526	.5213	.4910	.4617	.4336	.4066	.3807
.46E 00	.6292	.5966	.5647	.5337	.5037	.4746	.4465	.4195	.3935
.48E 00	.6401	.6080	.5765	.5458	.5160	.4871	.4591	.4321	.4061
.50E 00	.6505	.6189	.5879	.5575	.5279	.4992	.4713	.4444	.4184
.52E 00	.6606	.6295	.5988	.5688	.5395	.5110	.4833	.4564	.4305
.54E 00	.6703	.6397	.6095	.5798	.5508	.5225	.4949	.4682	.4423
.56E 00	.6796	.6495	.6197	.5904	.5617	.5336	.5063	.4796	.4538
.58E 00	.6887	.6590	.6297	.6008	.5724	.5445	.5173	.4908	.4651
.60E 00	.6974	.6682	.6393	.6108	.5827	.5551	.5281	.5018	.4761
.62E 00	.7058	.6771	.6486	.6205	.5927	.5654	.5386	.5124	.4869
.64E 00	.7139	.6857	.6577	.6299	.6025	.5754	.5489	.5229	.4975
.66E 00	.7217	.6940	.6664	.6391	.6120	.5852	.5589	.5331	.5078
.68E 00	.7293	.7021	.6749	.6480	.6212	.5947	.5687	.5431	.5179
.70E 00	.7366	.7099	.6832	.6566	.6302	.6040	.5782	.5528	.5279
.72E 00	.7437	.7175	.6912	.6650	.6389	.6131	.5875	.5623	.5375
.74E 00	.7506	.7248	.6990	.6731	.6474	.6219	.5966	.5716	.5470
.76E 00	.7573	.7319	.7065	.6811	.6557	.6305	.6055	.5807	.5563
.78E 00	.7637	.7388	.7138	.6888	.6638	.6389	.6141	.5896	.5654
.80E 00	.7699	.7455	.7210	.6963	.6716	.6470	.6226	.5983	.5743
.82E 00	.7760	.7520	.7279	.7036	.6793	.6550	.6308	.6068	.5831
.84E 00	.7819	.7584	.7346	.7107	.6867	.6628	.6389	.6151	.5916
.86E 00	.7876	.7645	.7411	.7176	.6940	.6704	.6468	.6233	.5999
.88E 00	.7931	.7704	.7475	.7244	.7011	.6778	.6545	.6312	.6081
.90E 00	.7984	.7762	.7537	.7309	.7080	.6850	.6620	.6390	.6161
.92E 00	.8036	.7818	.7597	.7373	.7147	.6920	.6693	.6466	.6240
.94E 00	.8087	.7873	.7655	.7435	.7213	.6989	.6765	.6540	.6317
.96E 00	.8136	.7926	.7712	.7496	.7277	.7056	.6835	.6613	.6392
.98E 00	.8183	.7977	.7767	.7554	.7339	.7122	.6903	.6684	.6465
.10E 01	.8230	.8027	.7821	.7612	.7400	.7186	.6970	.6754	.6537
.11E 01	.8442	.8258	.8070	.7878	.7682	.7484	.7283	.7080	.6876
.12E 01	.8626	.8459	.8288	.8112	.7932	.7748	.7562	.7372	.7181
.13E 01	.8786	.8635	.8479	.8318	.8153	.7984	.7811	.7635	.7456
.14E 01	.8926	.8790	.8648	.8501	.8349	.8194	.8034	.7870	.7704
.15E 01	.9049	.8926	.8797	.8663	.8524	.8381	.8233	.8082	.7927
.16E 01	.9157	.9045	.8928	.8806	.8679	.8548	.8412	.8272	.8128
.17E 01	.9252	.9151	.9045	.8934	.8818	.8697	.8572	.8443	.8310
.18E 01	.9336	.9244	.9148	.9047	.8941	.8831	.8716	.8597	.8474
.19E 01	.9410	.9327	.9240	.9148	.9051	.8950	.8845	.8735	.8622
.20E 01	.9475	.9400	.9321	.9237	.9149	.9057	.8961	.8860	.8755
.21E 01	.9533	.9465	.9393	.9317	.9237	.9153	.9064	.8972	.8876

Table 3 (cont'd)

Probabilities for selected quantiles and shape parameter values

τ \ γ	.55	.60	.65	.70	.75	.80	.85	.90	.95
.22E 01	.9584	.9523	.9458	.9389	.9316	.9239	.9158	.9073	.8984
.23E 01	.9629	.9574	.9515	.9453	.9386	.9316	.9242	.9164	.9083
.24E 01	.9669	.9620	.9566	.9509	.9449	.9385	.9317	.9246	.9171
.25E 01	.9705	.9660	.9612	.9560	.9505	.9447	.9385	.9320	.9251
.26E 01	.9737	.9696	.9652	.9606	.9556	.9503	.9446	.9386	.9323
.27E 01	.9765	.9728	.9689	.9646	.9601	.9553	.9501	.9446	.9389
.28E 01	.9790	.9757	.9721	.9683	.9641	.9597	.9550	.9501	.9448
.29E 01	.9812	.9782	.9750	.9715	.9678	.9638	.9595	.9549	.9501
.30E 01	.9832	.9805	.9776	.9744	.9710	.9674	.9635	.9593	.9549
.35E 01	.9904	.9888	.9870	.9851	.9830	.9807	.9783	.9756	.9728
.40E 01	.9944	.9935	.9924	.9912	.9900	.9886	.9870	.9854	.9836
.45E 01	.9968	.9962	.9956	.9948	.9941	.9932	.9922	.9912	.9901
.50E 01				.9970	.9965	.9959	.9954	.9947	.9940
.55E 01								.9968	.9964

τ \ γ	1.0	1.1	1.2	1.3	1.4	1.5
.10E-03	.0001					
.10E-02	.0010	.0005	.0002	.0001		
.10E-01	.0100	.0060	.0036	.0021	.0013	.0007
.10E 00	.0952	.0721	.0542	.0406	.0302	.0224
.12E 00	.1131	.0872	.0668	.0509	.0386	.0291
.14E 00	.1306	.1022	.0795	.0615	.0473	.0363
.16E 00	.1479	.1172	.0923	.0724	.0564	.0438
.18E 00	.1647	.1320	.1052	.0834	.0658	.0516
.20E 00	.1813	.1468	.1181	.0946	.0754	.0598
.22E 00	.1975	.1613	.1311	.1059	.0852	.0681
.24E 00	.2134	.1758	.1440	.1173	.0951	.0767
.26E 00	.2289	.1900	.1568	.1288	.1052	.0855
.28E 00	.2442	.2041	.1696	.1402	.1154	.1945
.30E 00	.2592	.2180	.1823	.1517	.1257	.1036
.32E 00	.2739	.2317	.1950	.1632	.1360	.1128
.34E 00	.2882	.2452	.2075	.1747	.1464	.1221
.36E 00	.3023	.2586	.2200	.1862	.1568	.1315
.38E 00	.3161	.2717	.2323	.1976	.1673	.1410
.40E 00	.3297	.2847	.2445	.2090	.1778	.1505
.42E 00	.3430	.2974	.2566	.2203	.1882	.1601
.44E 00	.3560	.3100	.2686	.2316	.1987	.1697
.46E 00	.3687	.3224	.2804	.2427	.2092	.1794
.48E 00	.3812	.3346	.2921	.2539	.2196	.1891
.50E 00	.3935	.3466	.3037	.2649	.2299	.1987
.52E 00	.4055	.3584	.3151	.2758	.2403	.2084
.54E 00	.4173	.3700	.3264	.2866	.2506	.2181
.56E 00	.4288	.3814	.3376	.2974	.2608	.2278
.58E 00	.4401	.3926	.3486	.3080	.2710	.2374
.60E 00	.4512	.4037	.3594	.3186	.2811	.2470
.62E 00	.4621	.4146	.3702	.3290	.2912	.2566
.64E 00	.4727	.4252	.3807	.3394	.3011	.2661

Table 3 (cont'd)

Probabilities for selected quantiles and shape parameter values

τ \ γ	1.0	1.1	1.2	1.3	1.4	1.5
.66E 00	.4831	.4358	.3912	.3496	.3111	.2756
.68E 00	.4934	.4461	.4015	.3597	.3209	.2851
.70E 00	.5034	.4563	.4116	.3697	.3306	.2945
.72E 00	.5132	.4662	.4216	.3796	.3403	.3038
.74E 00	.5229	.4761	.4315	.3894	.3499	.3131
.76E 00	.5323	.4857	.4412	.3990	.3594	.3223
.78E 00	.5416	.4952	.4508	.4085	.3688	.3315
.80E 00	.5507	.5045	.4602	.4180	.3781	.3406
.82E 00	.5596	.5137	.4695	.4273	.3873	.3496
.84E 00	.5683	.9997	.4786	.4365	.3964	.3586
.86E 00	.5768	.5315	.4876	.4455	.4054	.3675
.88E 00	.5852	.5402	.4965	.4545	.4144	.3763
.90E 00	.5934	.5487	.5053	.4633	.4232	.3851
.92E 00	.6015	.5571	.5139	.4721	.4319	.3937
.94E 00	.6094	.5653	.5223	.4807	.4406	.4023
.96E 00	.6171	.5734	.5307	.4891	.4491	.4108
.98E 00	.6247	.5814	.5389	.4975	.4576	.4192
.10E 01	.6321	.5892	.5470	.5058	.4659	.4276
.11E 01	.6671	.6262	.5854	.5453	.5061	.4681
.12E 01	.6988	.6599	.6209	.5821	.5439	.5064
.13E 01	.7275	.6907	.6536	.6163	.5792	.5425
.14E 01	.7534	.7188	.6835	.6479	.6121	.5765
.15E 01	.7769	.7444	.7111	.6771	.6428	.6084
.16E 01	.7981	.7677	.7363	.7041	.6713	.6382
.17E 01	.8173	.7890	.7594	.7290	.6978	.6660
.18E 01	.8347	.8083	.7806	.7519	.7223	.6920
.19E 01	.8504	.8259	.8000	.7729	.7449	.7161
.20E 01	.8647	.8419	.8177	.7923	.7659	.7385
.21E 01	.8775	.8564	.8339	.8101	.7852	.7593
.22E 01	.8892	.8696	.8487	.8265	.8031	.7786
.23E 01	.8997	.8816	.8622	.8414	.8195	.7965
.24E 01	.9093	.8926	.8745	.8552	.8346	.8130
.25E 01	.9179	.9025	.8858	.8678	.8485	.8282
.26E 01	.9257	.9115	.8960	.8793	.8613	.8423
.27E 01	.9328	.9197	.9054	.8898	.8731	.8553
.28E 01	.9392	.9271	.9139	.8995	.8839	.8672
.29E 01	.9450	.9339	.9217	.9083	.8938	.8782
.30E 01	.9502	.9400	.9287	.9164	.9029	.8884
.35E 01	.9698	.9632	.9557	.9474	.9382	.9281
.40E 01	.9817	.9774	.9726	.9670	.9609	.9540
.45E 01	.9889	.9862	.9830	.9794	.9753	.9707
.50E 01	.9933	.9915	.9895	.9872	.9845	.9814
.55E 01	.9959	.9948	.9935	.9920	.9903	.9883
.60E 01		.9968	.9960	.9951	.9939	.9926
.65E 01					.9962	.9954

Table 4.1

Algorithm input for quantile ratio computations for γ = 3 - 22 and selected probabilities

.68440903E-01	.13539709E&00	-.22276772E&00	-.32197761E&00	-.13366428E&00	PB 1
.70154770E-01	.14121096E&00	-.20739324E&00	-.24387948E&00	-.90781245E-01	PB 2
.71286686E-01	.14473000E&00	-.19696011E&00	-.19828904E&00	-.10239516E&00	PB 3
.72157844E-01	.14727630E&00	-.18882653E&00	-.16592193E&00	-.10873253E&00	PB 4
.72877158E-01	.14927665E&00	-.18207035E&00	-.14089894E&00	-.98523796E-01	PB 5
.73495800E-01	.15092637E&00	-.17624132E&00	-.12043555E&00	-.82914629E-01	PB 6
.74042243E-01	.15233050E&00	-.17107946E&00	-.10315707E&00	-.69598948E-01	PB 7
.74534106E-01	.15355322E&00	-.16642859E&00	-.88146949E-01	-.59247622E-01	PB 8
.74983064E-01	.15463582E&00	-.16218118E&00	-.74920111E-01	-.45638966E-01	PB 9
.75397306E-01	.15560697E&00	-.15826359E&00	-.63128816E-01	-.33664710E-01	PB10
.78466676E-01	.16201847E&00	-.12934751E&00	.14208264E-01	.52010895E-01	PB11
.80601493E-01	.16572263E&00	-.10949622E&00	.59031710E-01	.10588637E&00	PB12
.82310846E-01	.16827926E&00	-.93831376E-01	.90242719E-01	.14087334E&00	PB13
.83770337E-01	.17019135E&00	-.80647535E-01	.11409116E&00	.16383555E&00	PB14
.85063543E-01	.17168716E&00	-.69128131E-01	.13310291E&00	.18165051E&00	PB15
.86237339E-01	.17288987E&00	-.58815663E-01	.14875102E&00	.18987157E&00	PB16
.87320986E-01	.17387421E&00	-.49421235E-01	.16193758E&00	.19637927E&00	PB17
.88334038E-01	.17468793E&00	-.40751650E-01	.17320441E&00	.19992577E&00	PB18
.89290312E-01	.17536452E&00	-.32673356E-01	.18294635E&00	.19853831E&00	PB19
.97161781E-01	.17782912E&00	.29632223E-01	.23350478E&00	.12298471E&00	PB20
.10376223E&00	.17610695E&00	.75610992E-01	.24196094E&00	.33448469E-02	PB21
.11009157E&00	.17164814E&00	.11392607E&00	.22802780E&00	-.11724675E&00	PB22
.11663385E&00	.16447786E&00	.14736432E&00	.19684960E&00	-.22394455E&00	PB23
.12382559E&00	.15391561E&00	.17676890E&00	.14950527E&00	-.29816413E&00	PB24
.13228269E&00	.13830843E&00	.20150917E&00	.84872764E-01	-.32660397E&00	PB25
.14323056E&00	.11364561E&00	.21815568E&00	.57899927E-04	-.27950802E&00	PB26
.16034850E&00	.66692577E-01	.21173993E&00	-.10574522E&00	-.98492325E-01	PB27
.16282487E&00	.59175105E-01	.20779170E&00	-.11667903E&00	-.67457137E-01	PB28
.16556126E&00	.50678327E-01	.20259932E&00	-.12736135E&00	-.37576306E-01	PB29
.16862527E&00	.40937174E-01	.19578777E&00	-.13738096E&00	-.45268249E-02	PB30
.17211527E&00	.29562686E-01	.18677698E&00	-.14630390E&00	.35312979E-01	PB31
.17618246E&00	.15951663E-01	.17467069E&00	-.15341020E&00	.75727046E-01	PB32
.18107789E&00	-.90786713E-03	.15791095E&00	-.15695878E&00	.11480029E&00	PB33
.18726705E&00	-.22914627E-01	.13350043E&00	-.15382489E&00	.15296481E&00	PB34
.19578077E&00	-.54334491E-01	.94509002E-01	-.13618683E&00	.17322981E&00	PB35
.20984487E&00	-.10878365E&00	.17934449E-01	-.75570225E-01	.14698614E&00	PB36
.21193453E&00	-.11711488E&00	.54016368E-02	-.63591013E-01	.14537149E&00	PB37
.21425678E&00	-.12644214E&00	-.88664720E-02	-.49088761E-01	.12503749E&00	PB38
.21687243E&00	-.13702993E&00	-.25329808E-01	-.31925988E-01	.10896349E&00	PB39
.21987018E&00	-.14926875E&00	-.44702873E-01	-.10820995E-01	.88586113E-01	PB40

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Table 4.1 (cont'd)

Algorithm input for quantile ratio computations for γ = 3 - 22 and selected probabilities

.22338678E&00	-.16376294E&00	-.68085797E-01	.15607192E-01	.56194542E-01PB41
.22764995E&00	-.18152426E&00	-.97348141E-01	.50052843E-01	.18868993E-01PB42
.23308347E&00	-.20444935E&00	-.13602611E&00	.97684746E-01	-.44241221E-01P843
.24063008E&00	-.23679031E&00	-.19212919E&00	.16944486E&00	-.12515575E&00PB44
.25325921E&00	-.29211561E&00	-.29178592E&00	.30643882E&00	-.36690261E&00PB45
.20552448E&02	-.30696131E&01	-.26695220E&00	.60260049E-02	-.73553285E-04GB 1
.20432134E&02	-.24206541E&01	-.76621535E-01	-.34313134E-02	.12894464E-03GB 2
.20290268E&02	-.18825336E&01	.24652504E-01	-.49226591E-02	.67347837E-04GB 3
.20151358E&02	-.14298066E&01	.78158101E-01	-.38931192E-02	-.14304087E-04GB 4
.20022172E&02	-.10427771E&01	.10398B14E&00	-.22055723E-02	-.63564203E-04G8 5
.19904029E&02	-.70710676E&00	.11292547E&00	-.54602588E-03	-.78955B93E-04GB 6
.19796472E&02	-.41233718E&00	.11117399E&00	.83419816E-03	-.70415776E-04GB 7
.19698510E&02	-.15071553E&00	.10250214E&00	.18627997E-02	-.47728635E-04G8 8
.19609061E&02	.83611659E-01	.89292065E-01	.25372006E-02	-.18465294E-04GB 9
.19527109E&02	.29515448E&00	.73097500E-01	.28832504E-02	.96900150E-05GB10
.19451746E&02	.48743608E&00	.54965963E-01	.29433733E-02	.34073779E-04GB11
.19382188E&02	.66326397E&00	.35608267E-01	.27523257E-02	.52516051E-04GB12
.19317757E&02	.82490166E&00	.15518014E-01	.23470276E-02	.62342092E-04GB13
.19257874E&02	.97419274E&00	-.49549665E-02	.17588355E-02	.62925241E-04GB14
.19202042E&02	.11126633E&01	-.25566356E-01	.10165045E-02	.55079382E-04GB15
.19149829E&02	.12415842E&01	-.46139671E-01	.14464919E-03	.37900331E-04GB16
.1910086BE&02	.13620240E&01	-.66549905E-01	-.B3644394E-03	.142B707BE-04GB17
.19054831E&02	.14748976E&01	-.86703487E-01	-.1915B115E-02	-.90430397E-05GB18
.19011456E&02	.15809643E&01	-.10655572E&00	-.30623688E-02	-.50308501E-04GB19
.18970486E&02	.16809039E&01	-.12605538E&00	-.42717591E-02	-.98798473E-04GB20

Table 4.2

Algorithm input for quantile ratio computations for γ = 20 - 104 and selected probabilities

.99881322E-01	-.18871401E&00	-.26298174E&00	-.35992913E&00	.41539210E&00PB 1
.10188944E&00	-.18670548E&00	-.22319488E&00	-.24591418E&00	.19433831E&00PB 2
.10316677E&00	-.18521279E&00	-.19857915E&00	-.18144427E&00	.81915188E-01PB 3
.10412579E&00	-.18398494E&00	-.18046147E&00	-.13668371E&00	.14835979E-01PB 4
.10490290E&00	-.18292388E&00	-.16601550E&00	-.10290109E&00	-.23182667E-01PB 5
.10556114E&00	-.18197942E&00	-.15395222E&00	-.75935302E-01	-.64539807E-01PB 6
.10613511E&00	-.18112207E&00	-.14355844E&00	-.53765694E-01	-.79489226E-01PB 7
.10664597E&00	-.18033284E&00	-.13441132E&00	-.34582243E-01	-.94342556E-01PB 8
.10710766E&00	-.17959847E&00	-.12622735E&00	-.18128889E-01	-.11831225E&00PB 9
.10752986E&00	-.17890961E&00	-.11881266E&00	-.37047166E-02	-.13076010E&00PB10
.11055327E&00	-.17350259E&00	-.67729170E-01	.83634919E-01	-.16714673E&00PB11
.11255811E&00	-.16947210E&00	-.35862646E-01	.12771319E&00	-.17481535E&00PB12
.11411220E&00	-.16611222E&00	-.12295114E-01	.15459019E&00	-.14081216E&00PB13
.11540613E&00	-.16316170E&00	.65526337E-02	.17268325E&00	-.13942427E&00PB14
.11652894E&00	-.16049117E&00	.22328486E-01	.18548354E&00	-.11368517E&00PB15
.11752992E&00	-.15802537E&00	.35937668E-01	.19440329E&00	-.94569277E-01PB16
.11843945E&00	-.15571665E&00	.47921827E-01	.20074787E&00	-.66326419E-01PB17
.11927763E&00	-.15353221E&00	.58646772E-01	.20535825E&00	-.41079595E-01PB18
.12005855E&00	-.15144863E&00	.68354943E-01	.20822792E&00	-.28316703E-01PB19
.12615097E&00	-.13365297E&00	.13470538E&00	.19972784E&00	.13768539E&00PB20
.13087289E&00	-.11809116E&00	.17447321E&00	.16320573E&00	.20838211E&00PB21
.13513473E&00	-.10282062E&00	.20155322E&00	.11620535E&00	.25985371E&00PB22
.13931185E&00	-.86804248E-01	.22003818E&00	.63236123E-01	.28518445E&00PB23
.14367840E&00	-.69029590E-01	.23099400E&00	.63676307E-02	.25194664E&00PB24
.14856029E&00	-.48005425E-01	.23340690E&00	-.53432157E-01	.20312472E&00PB25
.15454652E&00	-.20720647E-01	.22289000E&00	-.11362826E&00	.11900741E&00PB26
.16331914E&00	.21881934E-01	.18280759E&00	-.16324941E&00	-.23193232E-01PB27
.16453983E&00	.28029184E-01	.17509468E&00	-.16586982E&00	-.31781405E-01PB28
.16587637E&00	.34816482E-01	.16608478E&00	-.16752668E&00	-.44200475E-01PB29
.16735834E&00	.42409042E-01	.15542836E&00	-.16812189E&00	-.79985776E-01PB30
.16902856E&00	.51047592E-01	.14260015E&00	-.16635372E&00	-.95712586E-01PB31
.17095256E&00	.61101655E-01	.12677782E&00	-.16220577E&00	-.11388992E&00PB32
.17323855E&00	.73184213E-01	.10658267E&00	-.15361155E&00	-.12405960E&00PB33
.17608580E&00	.88431180E-01	.79411330E-01	-.13762502E&00	-.13744437E&00PB34
.17993202E&00	.10935379E&00	.39336341E-01	-.10749182E&00	-.14380852E&00PB35
.18613103E&00	.14379800E&00	-.32746960E-01	-.38126106E-01	-.99810101E-01PB36
.18703746E&00	.14890406E&00	-.44009373E-01	-.26291367E-01	-.11335289E&00PB37
.18804071E&00	.15457542E&00	-.56686305E-01	-.11950671E-01	-.92908377E-01PB38
.18916572E&00	.16095924E&00	-.71149663E-01	.45311859E-02	-.72725656E-01PB39
.19044878E&00	.16827035E&00	-.87967974E-01	.24175104E-01	-.48737463E-01PB40

Table 4.2 (cont.d)

Algorithm input for quantile ratio computations for γ = 20 - 104 and selected probabilities

.19194563E&00	.17684048E&00	-.10801073E&00	.48035983E-01	-.25956831E-01PB41
.19374878E&00	.18722062E&00	-.13273934E&00	.78644621E-01	-.60990423E-02PB42
.19602958E&00	.20043641E&00	-.16491819E&00	.11992006E&00	.61549507E-01PB43
.19916699E&00	.21876769E&00	-.21077093E&00	.18107879E&00	.14878758E&00PB44
.20434456E&00	.24938679E&00	-.29026866E&00	.29253559E&00	.34977127E&00PB45

.18999065E&02	.20738106E&01	-.68135165E-01	.12961368E-02	.23119701E-04GB 1
.18976302E&02	.19603549E&01	-.56305560E-01	.81384021E-03	.30108261E-05GB 2
.18954790E&02	.18532210E&01	-.45935583E-01	.44497553E-03	-.34198469E-05GB 3
.18934417E&02	.17518204E&01	-.36819058E-01	.15884004E-03	-.61299187E-05GB 4
.18915083E&02	.16556487E&01	-.28793880E-01	-.60524384E-04	-.87819327E-05GB 5
.18896707E&02	.15642635E&01	-.21722285E-01	-.22491058E-03	-.98675034E-05GB 6
.18879213E&02	.14772703E&01	-.15488154E-01	-.34599073E-03	-.85963403E-05GB 7
.18862533E&02	.13943215E&01	-.99916449E-02	-.43111618E-03	-.83794802E-05GB 8
.18846607E&02	.13151072E&01	-.51463074E-02	-.48854118E-03	-.71589694E-05GB 9
.18831379E&02	.12393493E&01	-.87901114E-03	-.52244011E-03	-.56330222E-05GB10
.18816802E&02	.11667981E&01	.28743149E-02	-.53635992E-03	-.44050717E-05GB11
.18802828E&02	.10972293E&01	.61683989E-02	-.53802247E-03	-.26424267E-05GB12
.18789419E&02	.10304399E&01	.90522697E-02	-.52506773E-03	-.13134717E-05GB13
.18776538E&02	.96524544E&00	.11568022E-01	-.50510574E-03	-.31182231E-06GB14
.18764149E&02	.90447985E&00	.13754334E-01	-.47689888E-03	.12343753E-05GB15
.18752225E&02	.84499013E&00	.15642001E-01	-.44351639E-03	.21282077E-05GB16
.18740734E&02	.78763605E&00	.17261591E-01	-.40543123E-03	.30406620E-05GB17
.18729652E&02	.73229297E&00	.18638905E-01	-.36484835E-03	.42741631E-05GB18
.18718956E&02	.67884201E&00	.19796052E-01	-.32250204E-03	.46914046E-05GB19
.18708622E&02	.62717613E&00	.20754952E-01	-.27889388E-03	.47532576E-05GB20
.18698633E&02	.57719594E&00	.21533044E-01	-.23499671E-03	.49439462E-05GB21
.18688967E&02	.52880999E&00	.22146657E-01	-.19137359E-03	.53347816E-05GB22
.18679609E&02	.48193506E&00	.22610711E-01	-.14838210E-03	.59856252E-05GB23
.18670541E&02	.43649124E&00	.22939552E-01	-.10596038E-03	.54228773E-05GB24
.18661750E&02	.39240622E&00	.23143595E-01	-.64496935E-04	.58701783E-05GB25
.18653221E&02	.34961189E&00	.23234207E-01	-.24867369E-04	.55572522E-05GB26
.18644942E&02	.30804580E&00	.23221661E-01	.13091587E-04	.59489766E-05GB27
.18636900E&02	.26765047E&00	.23113638E-01	.49011571E-04	.50804721E-05GB28
.18629084E&02	.22836946E&00	.22920166E-01	.83448234E-04	.52830352E-05GB29
.18621483E&02	.19015123E&00	.22646488E-01	.11572759E-03	.43497967E-05GB30
.18614089E&02	.15295027E&00	.22300905E-01	.14493126E-03	.43158790E-05GB31
.18606891E&02	.11671922E&00	.21888698E-01	.17364570E-03	.33092842E-05GB32
.18599883E&02	.81416478E-01	.21415307E-01	.19959654E-03	.31207246E-05GB33
.18593053E&02	.47002147E-01	.20886193E-01	.22305861E-03	.26915513E-05GB34

Table 4.2 (cont'd)

Algorithm input for quantile ratio computations for γ = 20 - 104 and selected probabilities

.18586395E&02	.13439745E-01	.20305866E-01	.24510594E-03	.21583469E-05G835
.18579903E&02	-.19306074E-01	.19679166E-01	.26397330E-03	.20470787E-05G836
.18573570E&02	-.51268898E-01	.19008241E-01	.28165364E-03	.92244514E-06G837
.18567387E&02	-.82479030E-01	.18298787E-01	.29673328E-03	.10171327E-05GB38
.18561351E&02	-.11296662E&00	.17552489E-01	.30994707E-03	.77998907E-07GB39
.18555455E&02	-.14276071E&00	.16773661E-01	.32107044E-03	.18750778E-06GB40
.18549693E&02	-.17188550E&00	.15964756E-01	.32965615E-03	-.62910041E-06GB41
.18544061E&02	-.20036771E&00	.15128473E-01	.33733145E-03	-.97494543E-06GB42
.18538555E&02	-.22823018E&00	.14266167E-01	.34276756E-03	-.10595267E-05GB43
.18533168E&02	-.25549580E&00	.13380836E-01	.34559443E-03	-.19841424E-05GB44
.18527897E&02	-.28218563E&00	.12474427E-01	.34778615E-03	-.15192654E-05G845
.18522738E&02	-.30831967E&00	.11549105E-01	.34757733E-03	-.29958722E-05GB46
.18517685E&02	-.33391758E&00	.10605804E-01	.34556255E-03	-.32316795E-05GB47
.18512738E&02	-.35899796E&00	.96463758E-02	.34190650E-03	-.32446747E-05GB48
.18507891E&02	-.38357805E&00	.86730845E-02	.33742661E-03	-.38742677E-05GB49
.18503141E&02	-.40767434E&00	.76866761E-02	.33032437E-03	-.36535604E-05G850
.18498484E&02	-.43130352E&00	.66889109E-02	.32281659E-03	-.42123373E-05GB51
.18493919E&02	-.45447919E&00	.56805725E-02	.31308538E-03	-.43891298E-05GB52
.18489441E&02	-.47721687E&00	.46613861E-02	.30177421E-03	-.41639906E-05GB53
.18485047E&02	-.49953099E&00	.36352490E-02	.28854256E-03	-.52258372E-05GB54
.18480737E&02	-.52143367E&00	.26020571E-02	.27559960E-03	-.45318215E-05GB55
.18476506E&02	-.54293791E&00	.15611597E-02	.26070433E-03	-.48378551E-05GB56
.18472352E&02	-.56405532E&00	.51487073E-03	.24464666E-03	-.43110278E-05GB57
.18468273E&02	-.58479864E&00	-.53639520E-03	.22694679E-03	-.45734351E-05GB58
.18464266E&02	-.60517911E&00	-.15916296E-02	.20794633E-03	-.43503713E-05GB59
.18460331E&02	-.62520485E&00	-.26504975E-02	.18838341E-03	-.54167461E-05GB60
.18456463E&02	-.64488938E&00	-.37135574E-02	.16768788E-03	-.41590888E-05GB61
.18452662E&02	-.66423982E&00	-.47781385E-02	.14635781E-03	-.40387850E-05GB62
.18448925E&02	-.68326749E&00	-.58443939E-02	.12315929E-03	-.38887090E-05GB63
.18445251E&02	-.70197986E&00	-.69127801E-02	.99637903E-04	-.42579284E-05GB64
.18441638E&02	-.72038721E&00	-.79812405E-02	.74798507E-04	-.30548859E-05GB65
.18438085E&02	-.73849646E&00	-.90512067E-02	.49714917E-04	-.26947897E-05G866
.18434588E&02	-.75631660E&00	-.10120109E-01	.22911092E-04	-.28197255E-05G867
.18431148E&02	-.77385522E&00	-.11189053E-01	-.45763190E-05	-.26739235E-05GB68
.18427762E&02	-.79111959E&00	-.12257598E-01	-.32291580E-04	-.19806291E-05GB69
.18424429E&02	-.80811624E&00	-.13325568E-01	-.61108759E-04	-.18416764E-05GB70
.18421149E&02	-.82485279E&00	-.14392048E-01	-.90644380E-04	-.12646606E-05GB71
.18417918E&02	-.84133630E&00	-.15456397E-01	-.12111576E-03	-.44705458E-06GB72
.18414736E&02	-.85757285E&00	-.16520122E-01	-.15173021E-03	-.13418695E-06GB73
.18411602E&02	-.87356857E&00	-.17581398E-01	-.18339908E-03	-.12741381E-06GB74

+

Table 4.2 (cont'd)

Algorithm input for quantile ratio computations for γ = 20 - 104 and selected probabilities

.18408515E&02	-.88932830E&00	-.18640463E-01	-.21609588E-03	.87816205E-06GB75
.18405473E&02	-.90485979E&00	-.19696706E-01	-.24856010E-03	.13526246E-05GB76
.18402476E&02	-.92016741E&00	-.20750753E-01	-.28200894E-03	.14553698E-05GB77
.18399521E&02	-.93525639E&00	-.21801719E-01	-.31613981E-03	.31536027E-05GB78
.18396610E&02	-.95013255E&00	-.22849586E-01	-.35031696E-03	.37884509E-05GB79
.18393739E&02	-.96480107E&00	-.23893964E-01	-.38577629E-03	.40386764E-05GB80
.18390908E&02	-.97926653E&00	-.24936449E-01	-.42111498E-03	.52538316E-05GB81
.18388116E&02	-.99353275E&00	-.25974433E-01	-.45741560E-03	.60808736E-05GB82
.18385363E&02	-.10076050E&01	-.27008920E-01	-.49447153E-03	.66241490E-05GB83
.18382647E&02	-.10214894E&01	-.28039981E-01	-.53056210E-03	.78755355E-05GB84
.18379969E&02	-.10351866E&01	-.29067934E-01	-.56765876E-03	.88249211E-05GB85

Table 5. Table of critical values for the Kolmogorov-Smirnov Test. (a) Sample size 25, (b) sample size 30, and (c) asymptotic values. All asymptotic values are to be multiplied by $(1/N)^{\frac{1}{2}}$. All values adjusted to be no lower than normal where necessary.

Distributions		Probability Levels (α)					
		.20	.15	.10	.05	.01	N
Non-parametric	(a)	.208	.220	.238	.264	.317	25
No parameters	(b)	.190	.200	.218	.242	.290	30
Estimated	(c)	1.07	1.14	1.22	1.36	1.63	N
Exponential distribution	(a)	.170	.180	.191	.210	.247	25
Location parameter estimated	(b)	.155	.164	.174	.192	.226	30
Shape parameter 1.0 (gamma)	(c)	.86	.91	.96	1.06	1.25	N
Exponential distribution							
Shape parameter estimated	(a)	.165	.173	.185	.204	.241	25
and equal to 1	(b)	.152	.159	.169	.184	.214	30
Scale parameter estimated	(c)	.84	.89	.95	1.05	1.20	N
Gamma distribution							
Shape parameter estimated	(a)	.159	.166	.176	.190	.222	25
and equal to 2	(b)	.146	.153	.161	.175	.203	30
Scale parameter estimated	(c)	.81	.85	.91	.97	1.16	N
Gamma distribution							
Shape parameter estimated	(a)	.148	.156	.166	.180	.208	25
and equal to 3	(b)	.136	.143	.151	.165	.191	30
Scale parameter estimated	(c)	.77	.81	.86	.94	1.08	N
Gamma distribution							
Shape parameter estimated	(a)	.146	.154	.164	.178	.209	25
and equal to 4	(b)	.134	.140	.148	.163	.191	30
Scale parameter estimated	(c)	.75	.79	.83	.91	1.06	N
Gamma distribution							
Shape parameter estimated and	(a)	.143	.149	.159	.173	.203	25
equal to or greater than 8	(b)	.131	.137	.146	.161*	.186*	30
Scale parameter estimated	(c)	.74	.78	.81 *	.89 *	1.04	N
Gamma distribution							
Shape parameter known	(a)	.160	.167	.178	.194	.225	25
and greater than 3	(b)	.147	.154	.165	.180	.212	30
Scale parameter estimated	(c)	.800	.838	.893	.970	1.12	N
Normal distribution	(a)	.142	.147	.158	.173	.200	25
Location parameter estimated	(b)	.131	.136	.144	.161	.187	30
Scale parameter estimated	(c)	.736	.768	.805	.886	1.031	N
Extreme value distribution	(a)	.152	.157	.170	.183	.209	25
Location parameter estimated	(b)	.134	.140	.149	.164	.190	30
Scale parameter estimated	(c)	.738	.769	.816	.888	1.041	N

*adjusted to be no lower than the normal

Table 6

Plotting positions for ten reliability dewpoints
(from Kao, 1968)

i	1	2	3	4	5	6	7	8	9	10
x_i	2.75	3.1	3.4	3.8	4.1	4.4	4.7	5.1	5.7	6.4
$i/(n+1)$	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9	91.0

APPENDIX 2

USE OF ALGORITHM TO DETERMINE QUANTILE RATIO FOR A GIVEN SHAPE PARAMETER AND PROBABILITY

The following is an example which shows how to use the algorithm to determine a quantile ratio for a given shape parameter and probability.

Given γ = 10 and P = .5, the algorithm is

$$f(x,y) = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 + x_5y_5 .$$

The x_i and y_i values may be associated with the probability (PB) and gamma (GB) arrays, respectively, where subscripts denote the column number.

1. Determine the index value for the proper row in the probability array using the following table:

Index Value	Probability
PB1(1)PB9	.001(.001).009
PB10(1)PB18	.01(.01).09
PB19(1)PB27	.10(.10).90
PB28(1)PB36	.90(.01).99
PB37(1)PB45	.991(.001).999

2. Determine the index value for the proper row in the gamma array using k = γ-2 for table 1 and k = γ-19 for table 2.

For the values given above, the values from rows GB8 and PB23 are used. (Note: Values are printed in floating decimal format. For example, .1321541E&02 = 13.21541; .1321541E-02 = .001321541). The values from GB8 are:

19.698510	-.15071553	.10250214	.0018627997	-.000047728635

The values from PB23 are:

.11663385	.16447786	.14736432	.19684960	-.22394455

Now form the products of each pair of numbers. This now gives:

2.297513	-.024789	.015105	.000367	.000011

and summing these five values yields 2.288207. This value multiplied by the lower limit of the quantile for γ = 10 and P = 5 would yield an estimate of

the true quantile. The quantile lower limit is given by

$$\tau = [P \cdot \Gamma(\gamma+1)]^{1/\gamma}$$

where $\Gamma(11) = 3628800$. The value of τ, therefore, is $[(.5)(3628800)]^{.1} = [1814400]^{.1} = 4.22545327572$. Multiplying this value by the ratio computed yields 9.668712 for the desired quantile.

APPENDIX 3

FORTRAN IV ELECTRONIC COMPUTER PROGRAM
FOR APPLICATION OF THE GAMMA DISTRIBUTION FUNCTION TO DATA SETS
AND
COMPUTER OUTPUT TO MICROFILM GRAPHS

The following are comments for use of the gamma distribution program (FORTRAN IV). The user may go directly to the program for implementation. The program contains comment cards where deemed appropriate.

The comments are provided for those who wish to have a more general understanding of just what is required for program initiation. They will be helpful as references if difficulties are encountered either during the initiation or the running of the program.

There are four types of header (control) cards associated with any request. Two of these are always required; the remaining two are required only when the user chooses to define his own table of quantiles and probability levels.

Control card 1

Card col.	Name in Program	Meaning
1-2	II	This is the beginning period number of data set. Usually, this would be 01 if the first period is desired. Care should be taken when working with multiple data points per input record if one chooses to start with other than the first period on each data set. Positional association is used in this program. For example, suppose one has 20 years of weekly rainfall data in cards with 13 weeks of data contained on 1 card--therefore 4 cards per year comprising a data set. If the user chooses to define II = 12, he should make certain that fields 1-11 do not contain invalid punches (blanks are permissible). The data for this year should then fall into the 12th field of the input card. If one

		chooses to start with 14, the first card for weeks 1-13 will not be required, however the card number must be 2 since data storage is computed by index = (card no. - 1) * No. pts/card + pt # in this card $01 \leq II \leq 52$.
3-4	JJ	Ending period number $II \leq JJ \leq 52$
5-6	N	Number of quantile and probability levels to compute. If the standard set is chosen, N = 52; otherwise N is specified by the user. Note if N < 52, the user should define his own set since the first N values of the defined set would be used. $01 \leq N \leq 52$
7	ICOD	Code definition required by the program. If period totals of a quantity (i.e., weekly rainfall) are the input data, ICOD = 1. If parameter data are input (i.e., γ, β, $\overline{X}$), ICOD = 2. $1 \leq ICOD \leq 2$
8	I2	Coded as 1 if 2 period totals are required, otherwise blank or zero. $0 \leq I2 \leq 1$
9	I3	Coded as 1 if 3 period totals are required, otherwise blank or zero. $0 \leq I3 \leq 1$
10	ITAB	If the defined set of quantiles and probability levels are used, ITAB = 0; if the user specified the tables, ITAB = 1. If ITAB = 1, the tables are read under format specifications of F4.2. If ITAB = 2, the user may specify the tables and these will be read under F4.0. $0 \leq ITAB \leq 2$

11-13	K1	The number of years in the data sample. This number is checked by the program and, if incorrect, an appropriate error message is printed. $001 \leq K1 \leq 999$
14-77	ASTN	64-character heading of the user's choice to appear at the top of each output page.
78	IFACT	IFACT = 1 if the user wishes to compute the quantiles by $\overline{X}/N$ where $\overline{X}$ is the gamma distribution mean for an individual period and N is defined in col. 5-6 above. Note that the user may or may not use the defined tables as provided by the program. If he does, then ITAB = 0 (col. 10) and IFACT = 1. If the user wants to use less than 52 levels, he must include a card for the quantity levels even though they will be overlaid by this option. $0 \leq IFACT \leq 1$
79	IA	If IA = 0, alpha (origin) is assumed to be zero. If IA = 1, alpha is defined by control card 4, col. 9-16. If IA = 2, alpha is computed by the program.
80	ICN	Coded for card recognition.

Control card 2

Card col.	Name in Program	Meaning
1-4	P(1)	Quantile levels in format of F4.2 if ITAB = 1 or are in F4.0 if ITAB = 2. Note this card is not required if ITAB = 0.
5-8	P(2)	
.	.	
.	.	
.	.	
77-80	P(K)	
1-4	P(K=1)	If $20 < N \leq 40$, then a second card is needed. If $40 < N \leq 52$, a third card is required.
.	.	
.	.	
.	.	
	P(N)	

Control card 3

Card col.	Name in Program	Meaning
1-4	PL(1)	Probability levels in format of F4.2. Note this card is not required if ITAB = 0. The same conditions hold for the number of cards or in the quantile definition above.
5-8	PL(2)	
.	.	
.	.	
.	.	
77-80	PL(K)	
1-4	PL(K+1)	

Control card 4

Card col.	Name in Program	Meaning
1-2	IP	IP = number of data points contained on each individual card.
3-4	INT	INT = number of levels of chi-square grouping. If blank or zero, a default of 10 is used.
5-8	C	C = constant for computation of empirical probabilities. Default of 0.44 is used if C is blank or zero.
9-16	ALPHA	Origin definition if IA in card 1 is set to 1. If IA = 0, leave ALPHA blank or zero.
17-24	AJJ	AJJ is the largest value in the data set entry that is used by the program to detect missing data. For example, in the case of precipitation, if one week of data for a particular year was missing and the user had coded the missing value 99.99, then AJJ should be coded 99.99. AJJ is read under format specification F8.2. It should be noted that, if the user requested 2 or 3 period totals and the case of missing data were encountered with 99.99 defined for missing, the output would show an entry in the affected period that is greater than the 99.99; however, this would be omitted by the test of >99.99.
25-26	LIMIT	LIMIT is the controlling iteration value. If blank or zero, the default value of 10 is chosen. LIMIT is not used in the current version of the Program.

27-73	AFMT	AFMT is the user defined data format. Example of period total, 13 values/card and ICOD = 1. (I5, I2, I1, 13F4.2) I5 - STN or data set identifier in col. 1-5. I2 - Year of sequence number in col. 6-7. I1 - Card number within sequence # in col. 8. 13F4.2 - Thirteen fields of data with each field 4 cols. in width and an assumed decimal for data recorded to the nearest 0.01. Example for ICOD = 2. (I5, I2, I2, I3, I3, F6.2, F6.2, F6.2) I5 - Data set identifier in col. 1-5. I2 - Period number I col. 6-7. I2 - No. of weeks in period J in col. 8-9. I3 - NX = No. of years of nonzero entries, col. 10-12. I3 - NNX = No. of total years, col. 13-15. F6.2 - XBAR = gamma distribution mean, col. 16-21. F6.2 - GAMMA = shape parameter, col. 22-27. F6.2 - BETA = scale parameter, col. 28-33.
74	MILL	Input data are not in inches but millimeters (mm) and user wants quantiles converted to mm, code MILL = 1, otherwise 0.
75-80	BLANK	Not used.

If the user has multiple data sets to run with uniform characteristics (i.e., all options identical), only one control set of cards is required. Two blank cards terminate the run. If the data sets are different (for example, the number of years in one set is different from the remainder and requires its own set of control cards), then one blank card should be used to separate stations run under different controls.

```
      PROGRAM POA04T
C
C  POA04 IS A FORTRAN IV COMPUTER PROGRAM WRITTEN AT THE NATIONAL CLIMATIC
C  CENTER TO COMPUTE PRECIPITATION AMOUNTS AND/OR PROBABILITIES FROM PERIOD
C  TOTALS.
C CONTACT GRADY MCKAY, HAROLO CRUTCHER OR OAN FULBRIGHT AT THE NATIONAL
C CLIMATIC CENTER, ASHEVILLE, NC IF QUESTIONS ARISE ABOUT THE USE OF
C THIS PROGRAM.  IF THE PROGRAM IS USEO WITH THE RESULTS PUBLISHED, PLEASE
C MAKE APPROPRIATE ACKNOWLEDGEMENT.
C  THE PROGRAM IS WRITTEN TO ALLOW AS MUCH LATITUOE AS POSSIBLE FOR THE USER
C  BY PERMITTING BY CONTROL CARD SPECIFICATION THE FOLLOWING:
C    1 DEFINITION OF BEGINNING AND ENDING PERIOD
C    2 SELECTION OF THE NUMBER OF LEVELS TO COMPUTE
C    3 ALLOWING INPUT TO BE FROM PRE-COMPUTED PARAMETERS
C    4 ALLOWING OPTION OF COMPUTING TWO PERIOD OR THREE PERIOD STATISTICS
C    5 ALLOWING THE USER TO SPECIFY LEVELS OF QUANTILES AND PROBABILITIES OR
C      TO USE A PRE-DEFINED SET
C    6 ALLOW THE USER TO OEFINE THE HEADER LINE APPEARING ON THE OUTPUT
C    7 ALLOW THE USER TO SPECIFY THE ORIGIN UR PERMIT ITS COMPUTATION IF UNKNOWN
C    8 ALLOW THE USER TO SPECIFY THE NUMBER OF INPUT DATA POINTS PER RECORD
C      AND THEIR FORMAT AT RUN TIME
C    9 ALLOW THE USER TO SPECIFY THE NUMBER OF INTERVALS OF OATA GROUPING
C      FOR COMPUTATION OF CHI-SQUARE
C   10 ALLOW THE USER TO SPECIFY THE CONSTANT TO BE USED IN COMPUTING THE
C      EMPIRICAL PROBABILITIES
C
C  THE REQUIREO ANO OPTIONAL CONTROL CARDS ARE AS FOLLOWS
C  FIRST CONTROL CARD
C  POSITION   NAME    DEFINITION
C  1-2        II      BEGINNING PERIOO NUMBER
C  3-4        JJ      ENDING PERIOD NUMBER
C  5-6        N       NUMBER OF QUANTILE AND PROBABILITY LEVELS TO COMPUT
C  7          ICOD    CODED 1 IF PERIOD TOTALS TO BE USED. COOEO 2 FOR PARAMETERS
C  8          I2      CODEO 1 IF TWO PERIOD TOTALS ARE REQUIRED OTHERWISE BLANK
C  9          I3      CODEO 1 IF 3 PERIOD TOTALS ARE REQUIRED OTHERWISE BLANK
C  10         ITAB    CODEO 0 FOR DEFINED TABLES, 1 IF YOU SPECIFY TABLES
C CODE 2 IF DATA ARE IN MM AND QUANTILES ARE IN MM
C  11-13      K1      NUMBER OF YEARS IN OATA SAMPLE
C  14-77      ASTN    64 CHARACTER HEADING LINE
C  78         IFACT   CODE 1 IF PRECIP LEVELS ARE TO BE DIFINED BY XBAR/N
C  79         IA      USER WILL SUPPLY ALPHA IF CODED 1, 0 MEANS ALPHA=0,
C                     2 REQUIRES COMPUTATION OF ALPHA
C  80         ICN     CODED 1 FOR CARO RECOGNITION
C
C  IF  ITAB=1 ON THE PRECEEDING CARD THEN N VALUES OF QUANTILES AND
C  PROBABILITIES MUST BE REAO. THESE ARE REAO UNDER FORMAT CONTROL OF
C  20F4.2   QUANTILES ARE READ FIRST WITH AS MANY CARDS USED AS ARE
C  REQUIRED TO CONTAIN N VALUES. PROBABILITIES ARE THEN REAO IN THE SAME
C  FASHION. (NOTE THESE ARE A SEPARATE SET. UNUSED PORTIONS OF THE QUANTILE
C  CARD CAN NOT BE USEO TO DEFINE PROBABILITIES)
```



```
C
C  THE NEXT HEAOER CARD IS AS FOLLOWS:
C  POSITION   NAME   DEFINITION
C  1-2        IP     NUMBER OF DATA POINTS PER RECORD
C  3-4        INT    NUMBER OF INTERVALS FOR GROUPS (CHI-SQ.) OEFAULT=10 IF BLK
C  5-8        C      CONSTANT FOR EMPIRICAL PROB  OEFAULT=.44 IF BLANK
C  9-16       ALPHA  ORIGIN IF KNOWN AND IF IA ON PREVIOUS RECORO IS 1
C  17-24      AJJ    LARGEST VALUE TO BE ASSIGNED FOR MISSING OATA   F8.2
C  25-26      LIMIT  CONTROLLING ITERATION FACTOR  I2
C  27-74      AFMT   USER DEFINED OATA FORMAT
C  75         MILL   INPUT VALUES ARE IN MM ANO QUANTILES SHOULD BE CONVERTED TO
C                    MM EQUIVALENT  I1 1=YES BLANK OR ZERO =NO
C  76         J1=1 IF OENSITY CURVE PLOT REQUESTED
C  77         J2=1 IF HISTOGRAM CURVE PLOT REQUESTED
C  78         J3=1 IF CUMULATIVE FREQUENCY PLOT REQUESTEO
C  79         J4=1 IF CUMULATIVE MODEL FIT PLOT REQUESTED
C
C  INPUT EQUIPMENT CONSISTS OF A CARO READER  OEVICE 2
C  OUTPUT IS THE LINE PRINTER  DEVICE 6
C  TAPE OUTPUT ON TAPE 7 IS THE SAME INFORMATION AS WRITTEN ON THE PRINTER
C
C
C
C  DEFINITION OF ALL STOP INSTRUCTIONS
C  1111 ENO OF PROGRAM
C  2222 HEAOER CARD COUNT OF NUMBER OF YEARS DISAGREES WITH DATA COUNT
C  3333 DIFICULTY WITH CHI-SQ FIT
C  4444 DIFICULTY WITH PROBABILITY FIT
C  5555 UNABLE TO COMPUTE PROBABILITIES
C
C
C  STORAGE ALLOCATION IS AS FOLLOWS:
C   X(100) TEMPORARY REAO IN AREA
C   X1(3900)  INPUT DATA BY WEEK (OR MONTH) FOR 1 PERIOD TOTALS
C   X2(3900)  INPUT DATA BY WEEK (OR MONTH) FOR 2 PERIOD TOTALS
C   X3(3900)  INPUT DATA BY WEEK (OR MONTH) FOR 3 PERIOD TOTALS
C   D(100)    ORDERED OATA SET FOR CURRENT PERIOD
C   EMP(100)  EMPIRICAL PROB FOR CURRENT PERIOD
C   P1(100)   QUANTILES FOR EMPIRICAL PROB WITH BETA=1
C   P2(100)   QUANTILES FOR EMPIRICAL PROB WITH BETA=BETA
C   P3(52)    QUANTILES FOR SELECTED PROB LEVELS WITH BETA=1
C   P4(52)    QUANTILES FOR SELECTED PROB LEVELS WITH BETA=BETA
C   P5(52)    PROBABILITIES FOR SELECTED QUANTILES
C   P6(52)    GRAPH PROB FOR SELECTED QUANTILES (FOR X>0)
C   P(52)     SELECTED QUANTILE LEVELS
C   PL(52)    SELECTED PROBABILITY LEVELS
C   ATAB1(52) PRE-OEFINED QUANTILE TABLE
C   ATAB2(52) PRE-OEFINED PROB TABLE
C
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
```

```
      COMMON X(100),X1(3900),X2(3900),X3(3900),D(100),EMP(100),AJJ
      COMMON P1(100),P2(100),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
      COMMON LIMIT,IFACT
      DIMENSION TEM(15),ATAB1(52),ATAB2(52),AFMT(6)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 X,X1,X2,X3,D,AJJ,P1,P2,P3,P4,P5,P6,P,PL,ATAB1,ATAB2
      REAL*4 EMP,H
      REAL*4 CHI
      DATA  ATAB1/.005,.010,.015,.020,.025,.030,.035,.040,.045,.050,.060
     1,.070,.080,.090,.100,.15,.20,.25,.30,.35,.40,.45,.50,.55,.60,.65,
     2.70,.75,.80,.85,.90,.95,1.0,1.5,2.0,2.5,3.0,3.5,4.0,4.5,5.0,5.5,
     36.0,7.0,8.0,9.0,10.0,15.0,20.0,25.0,30.0,35.0/
      DATA ATAB2/.001,.003,.005,.006,.007,.008,.009,.010,.015,.020,.025,
     1             .030,.035,.040,.045,.050,.055,.060,.065,.070,.075,.080,
     2             .085,.090,.095,.100,.150,.200,.250,.300,.350,.400,.450,
     3             .500,.550,.600,.631,.650,.700,.750,.800,.850,.900,.910,
     4             .930,.950,.970,.980,.990,.995,.997,.999/
      REWIND 7
C  READ FIRST CONTROL CARD
  15  READ (2,1,END=9) II,JJ,N,ICOD,I2,I3,ITAB,K1,ASTN,IFACT,IA,ICN
 1    FORMAT  (3I2,4I1,I3,8A8,3I1)
C CLEAR SUMS AREAS TO ZERO
      DO 11 M=1,100
      X(M)=0.
      D(M)=0.
      EMP(M)=0.
      P1(M)=0.
      P2(M)=0.
  11  CONTINUE
      DO 12 M=1,3900
      X1(M)=0.
      X2(M)=0.
      X3(M)=0.
  12  CONTINUE
C CLEAR PRECIP AND PROB TABLES TO ZERO.
      DO 14 M=1,52
      P3(M)=0.
      P4(M)=0.
      P5(M)=0.
      P6(M)=0.
 14   CONTINUE
      IPLOT=0
      ALPHA=0.
C  IS THIS THE FIRST CONTROL CARD OF A GROUP? IF NOT PRINT AND STOP
      IF (ICN.EQ.1) GO TO 16
      WRITE (6,13) II,JJ,N,ICOD,I2,I3,ITAB,K1,ASTN,IFACT,IA,ICN
 13   FORMAT ('1',3I2,4I1,I3,8A8,3I1)
C     WRITE  (6,2)
 9    WRITE  (6,2)
```



```
 2    FORMAT (1H1,' GAMMA PROGRAM IS FINISHED'////)
      ENDFILE 7
      REWIND 7
      IF (IPLOT.EQ.0) GO TO 10
      CALL CALCMP (0.,0.,9999,2)
 10   STOP 1111
16    IF (ITAB.EQ.0) GO TO 18
C ITAB=0 IMPLIES THE USE OF DEFINED VALUES OF PRECIP AND PROBABILITIES.
      IF (ITAB.EQ.2) GO TO 17
      READ (2,3) (P(I),I=1,N)
 3    FORMAT (20F4.2)
      READ (2,3) (PL(I),I=1,N)
      GO TO  20
C IF ITAB=2, DATA AND QUANTILES ARE CODED IN MM
 6    FORMAT (20F4.0)
 17   READ (2,6) (P(I),I=1,N)
      READ (2,3) (PL(I),I=1,N)
      GO TO 20
18    DO 19 I=1,52
      P(I)=ATAB1(I)
19    PL(I)=ATAB2(I)
C  READ NEXT CONTROL CARD
 20   READ (2,4)  IP,INT,C,ALPHA,AJJ,LIMIT,AFMT,MILL,J1,J2,J3,J4
4     FORMAT (2I2,F4.2,F8.5,F8.2,I2,6A8,5I1)
      IF (J1.EQ.1.OR.J2.EQ.1.OR.J3.EQ.1.OR.J4.EQ.1) IPLOT=1
C  AFMT IS THE USER DEFINED OBJECT TIME FORMAT
C   SET DEFAULT VALUE IF UNDEFINED FOR C
      IF (C.EQ.0) C=.44
C   SET DEFAULT VALUE IF UNDEFINED FOR INT
      IF (INT.EQ.0) INT=10
C SET DEFAULT VALUE IF UNDEFINED FOR ALPHA
      IF (IA.EQ.0) ALPHA=0.
C SET DEFAULT VALUE IF LIMIT IS ZERO OR BLANK
      IF (LIMIT.EQ.0) LIMIT=10
      WRITE (6,5) II,JJ,N,ICOD,I2,I3,ITAB,K1,C,IA,LIMIT,IPLOT
 5    FORMAT (1H1,' THE FOLLOWING CONTROL PARAMETERS HAVE BEEN READ'///
     1' BEGINNING PERIOD NUMBER IS ',24X,I2/
     2' ENDING PERIOD NUMBER IS ',27X,I2/
     3' NUMBER OF PRECIP AND/OR PROB LEVELS IS ',12X,I2/
     4' OPTION IS ',41X,I2/
     5' TWO PERIOD TOTALS REQUIRED, YES IF NON-ZERO ',07X,I2/
     6' THREE PERIOD TOTALS REQUIRED, YES IF NON-ZERO ',05X,I2/
     7' USE DEFINED TABLES OF PRECIP OR PROB, YES IF 0 ',4X,I2/
     8' NUMBER OF YEARS USED IS ',26X,I3/
     9' C VALUE USED IS ',33X,F4.2/
     9' ALPHA VALUE TO BE COMPUTED IF IA=2,       '    ,10X,I2/
     9' VALUE TO CONTROL ITERATION LIMIT = ',16X,I2/
     9' COM GRAPHS REQUESTED IF IPLOT=1 ',19X,I2)
C COMPUTE INTERVAL VALUES FOR CHI SQUARE COMPUTATIONS  (K INTERVALS)
      L=INT-1
```

```
      V=INT
      DO 35 I=1,L
      H(I)= I/V
   35 CONTINUE
C SHOULD THE QUANTILE TABLE BE CONVERTED FROM INCHES TO MM
      IF (MILL.EQ.0) GO TO 37
      DO 36 I=1,N
   36 P(I)=P(I)*25.4
C  ICOD=2 IMPLIES USE OF PRE-COMPUTED PARAMETER VALUES.
   37 IF (ICOD.EQ.2) GO TO 55
C WE WILL READ ALL INPUT DATA AND STORE AWAY INTO 1, 2 AND 3 PERIOD TOTALS
C  SET NUM=0 FOR CHECKING THE ACTUAL NUMBER OF YEARS OF DATA TO BE READ
      NUM=0
      READ (2,AFMT) ID1,IY,IC,(TEM(J),J=1,IP)
C  ID1= IDENTIFICATION OF THIS SAMPLE
C  IY = YEAR NUMBER
C  IC = CARD NUMBER WITHIN YEAR
C  TEM= DATA POINTS  (IP OF THEM).
C
   44 NUM=NUM+1
   45 DO  46 I=1,IP
C COMPUTE POINT NUMBER FOR DATA ARRAY
      J=(IC-1)*IP+I
   46 X(J)=TEM(I)
      READ (2,AFMT) ID2,IY1,IC,(TEM(J),J=1,IP)
      IF (IY.EQ.IY1) GO TO 45
C AT THIS POINT WE HAVE READ A COMPLETE DATA YEAR AND HAVE STORED VALUES IN X(I)
C
C K1 EQUALS TOTAL NUMBER OF YEARS TO PROCESS
      DO 50 J=II,JJ
      I=(J-1)*K1+NUM
C  DATA ARE STORED ALL YEARS BY WEEK (OR MONTH) NUMBER FOR DURATION PERIOD 1
      X1(I)=X(J)
      IF (I2.EQ.0.AND.I3.EQ.0) GO TO 50
      K=J+1
      L=J+2
      IF (J.EQ.JJ) GO TO 47
C  DATA ARE STORED ALL YEARS BY WEEK (OR MONTH) NUMBER FOR DURATION PERIOD 2
      X2(I)=X(J)+X(K)
      IF (J.EQ.(JJ-1)) GO TO 48
C  DATA ARE STORED ALL YEARS BY WEEK (OR MONTH) NUMBER FOR DURATION PERIOD 3
      X3(I)=X(J)+X(K)+X(L)
      GO TO 50
   47 IF (ID1.NE.ID2) GO TC 50
      X2(I)=X(J)+TEM(1)
      X3(I)=X(J)+TEM(1)+TEM(2)
      CO TO 50
   48 IF (ID1.NE.ID2) GO TO 50
      X3(I)=X(J)+X(K)+TEM(1)
   50 CONTINUE
```



```
C
C WE NOW HAVE DATA STORED YEARS IN SUCCESSION BY WEEK NO. FOR  1, 2 AND  3 DUR
C PERIODS
      IY=IY1
      IF  (ID2.EQ.ID1) GO TO 44
C ALL  DATA FOR THIS STATION HAVE NOW BEEN READ.
      IF (NUM.EQ.K1) GO TO 60
C  SOMETHING WRONG LETS TAKE A LOOK.
      WRITE (6,51)  K1,NUM
   51 FORMAT (1H1,' YOU INDICATED ',I5,' WERE TO BE USED, HOWEVER OUR CO
     2UNT OF YEARS READ IS',I5///)
      STOP 2222
C
C
C
C  DATA TO BE READ FOR CONDITION ICOD=2, EACH CARD CONSTITUTES 1 DATA SET
   55 READ (2,AFMT,END=9) ID1,I,J,NX,NNX,XBAR,GAMMA,BETA
      IF (ID1.EQ.0) GO TO 15
C  COMPUTE PROB FOR SELECTED QUANTILES
      CALL  COMPUT (ICOD,P,P5,N)
      CALL COMPUT (99,P,P6,N)
      CALL  PRINT  (X1,I,J,ICOD)
      IF (IPLOT.NE.1) GO TO 55
      CALL PLOT (CHI,INT,GAMMA,D,BETA,P,P6,P4,PL,ASTN,ID1,H,P1,I,J,J1,
     1J2,J3,J4,AJJ)
      GO TO 55
C
   60 DO 100  KOUNT=1,3
      GO TO (58,56,57),KOUNT
C  CHECK FOR NEED OF 2 DURATION PERIODS
   56 IF (I2-0) 58,100,58
C  CHECK FOR NEED OF 3 DURATION PERIODS
   57 IF (I3-0) 58,100,58
C  COMPUTE PARAMETERS FOR 1 DURATION PERIOD
   58 DO  75   I=II,JJ
      GO TO (61,62,63),KOUNT
   61 CALL SUM (X1,I,D,CHI,ICOD)
      CALL COMPUT (ICOD,P,P5,N)
      CALL COMPUT (99,P,P6,N)
      CALL PRINT (X1,I,1,ICOD)
      IF (IPLOT.NE.1) GO TO 75
      CALL PLOT (CHI,INT.GAMMA,D,BETA,P,P6,P4,PL,ASTN,ID1,H,P1,I,1,J1,
     1J2,J3,J4,AJJ)
      GO TO 75
C  COMPUTE PARAMETERS FOR 2 DURATION PERIODS
   62 CALL SUM (X2,I,D,CHI,ICOD)
      CALL COMPUT (ICOD,P,P5,N)
      CALL COMPUT (99,P,P6,N)
      CALL PRINT  (X2,I,2,ICOD)
      IF (IPLOT.NE.1) GO TO 75
```

```
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          CALL PLOT (CHI,INT,GAMMA,O,BETA,P,P6,P4,PL,ASTN,ID1,H,P1,I,2,J1,
         1J2,J3,J4,AJJ)
          GO TO 75
C  COMPUTE PARAMETERS FOR 3 DURATION PERIODS
   63     CALL SUM (X3,I,O,CHI,ICOD)
          CALL COMPUT (ICOD,P,P5,N)
          CALL COMPUT (99,P,P6,N)
          CALL PRINT (X3,I,3,ICOD)
          IF (IPLOT.NE.1) GO TO 75
          CALL PLOT (CHI,INT,GAMMA,O,BETA,P,P6,P4,PL,ASTN,ID1,H,P1,I,3,J1,
         1J2,J3,J4,AJJ)
   75     CONTINUE
  100     CONTINUE
          ID1=ID2
          IY=IY1
          NUM=1
          IF (ID2.NE.0) GO TO 45
          GO TO 15
          END
```

```
FORTRAN IVL27 SOURCE PROGRAM  SUM      SUBROUTINE           03/25/77   PAGE 0008

          SUBROUTINE SUM (Y,I,O,CHI,ICOD)
C
C  THE SUM SUBROUTINE DOES THE FOLLOWING
C     1 SELECTS THE ITH WEEK FROM THE APPROPRIATE DURATION PERIOD
C     2 SORTS SELECTED DATA IN ASCENDING ORDER OF MAGNITUDE
C     3 IF REQUIRED COMPUTES ORIGIN ALPHA
C     4 COMPUTES SUMS AND LOGS FOR COMPUTATION OF PARAMETERS
C
          COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
          COMMON X(100),X1(3900),X2(3900),X3(3900),O(100),EMP(100),AJJ
          COMMON P1(100),P2(100),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
          COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
          COMMON LIMIT,IFACT
          DIMENSION Y(1),TEM(50),T(50)
          IMPLICIT REAL*8 (A-H,O-Z)
          REAL*4 X,X1,X2,X3,O,AJJ,P1,P2,P3,P4,P5,P6,P,PL,Y
          REAL*4 EMP,H
          REAL*4 CHI
C  IF ICOD=2 GO TO RETURN SINCE PARAMETERS ARE ALREADY AVAILABLE
          IF (ICOD.EQ.2) GO TO 70
          SX=0.
          SLX=0.
          NX=0
          NNX=0
          DO 7  J=1,NUM
    7     O(J)=AJJ
C  COMPUTE BEGIN AND END OF STORAGE FOR DATA SELECTION
          M=I*NUM-(NUM-1)
          NN=M+NUM-1
          V=0.
          K=1
C  MOVE SELECTED DATA INTO O
          DO 8 J=M,NN
          IF (Y(J).GE.AJJ) GO TO 8
          O(K)=Y(J)
          K=K+1
          IF (Y(J).GT.0.) V=V+1.
    8     CONTINUE
          IF (K.EQ.1) GO TO 70
          K=K-1
C  SORT DATA IN O INTO ASCENDING ORDER
          DO 25 L1=1,K
          DO 25 L2=L1,K
          IF (O(L2).GT.O(L1))  GO TO 25
          QQ=O(L2)
          O(L2)=O(L1)
          O(L1)=QQ
   25     CONTINUE
C  IF IA=1, ALPHA WAS SPECIFIED IN THE HEADER CARD
          IF (IA.LT.2) GO TO 9
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C  IF THE LEAST VALUE OF D IS ZERO, THE ORIGIN IS DEFINED
      IF (D(1).EQ.0.) GO TO 6
C  IF AT LEAST 2 VALUES (SMALLEST) ARE EQUAL ALPHA IS MADE THIS VALUE
      IF (D(1).EQ.D(2)) GO TO 6
C  ALPHA IS MADE SLIGHTLY SMALLER THAN THE SMALLEST ENTRY
      ALPHA=D(1)-D(1)*.00001
      GO TO 9
6     ALPHA=D(1)
9     NX=0
      SX=0.
      SLX=0.
C  COMPUTE SUMS AND LOGS FOR THE NON-ZERO ENTRIES
      DO 20 J=M,NN
      IF (Y(J).GE.AJJ ) GO TO 20
      IF ((Y(J)-ALPHA).LE.0.) GO TO 5
      SX=SX+Y(J)-ALPHA
      SLX=SLX+DLOG(Y(J)-ALPHA)
      NX=NX+1
5     NNX=NNX+1
20    CONTINUE
      IF (NX.LT.6) GO TO 70
C  COMPUTE CHI-SQUARE
      L=INT-1
      XBAR=SX/NX
      D=4.*(DLOG(XBAR)-SLX/NX)
      GAMMA=(1.+SQRT(1.+D/3.))/D
      BETA=XBAR/GAMMA
      CALL GAMIT(1)
      CALL INVGAM (BETA,H,CHI,L,NX,NX)
      DO 26 J=1,50
      TEM(J)=0.
      T(J)=0.
26    CONTINUE
      M=0
      K=1
      L=0
      DO 40 J=1,NNX
      IF (D(J).EQ.0.) GO TO 40
27    IF (D(J).LE.CHI(K)) GO TO 30
      T(K)=M
      TEM(K)=L
      K=K+1
      L=0
      IF (K-INT) 27,75,80
 30   M=M+1
      L=L+1
40    CONTINUE
45    DEN=NX/FLOAT(INT)
      SS =0.
      DO 50 L=1,INT
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      SS =SS + (TEM(L)-DEN)*(TEM(L)-DEN)
50    CONTINUE
      QQ=SS /DEN*.5
      GAMMA= (INT-3)*.5
      BETA=1.
      IF (IFACT.EQ.0) GO TO 51
      P(1)=10.*XBAR/N
      PART=P(1)
      DO 52 L=2,N
      P(L)=P(L-1)+PART
52    CONTINUE
C  COMPUTE PROBABILITY OF CHI-SQUARE
51    CALL COMPUT (99,QQ,PROB,1)
C  COMPUTE S-K
      QQ=QQ+QQ
      DIFF=-9999.
      SS =0.
      DO 65 L=1,K
      SS=SS+DEN
      CHECK=ABS(T(L)-SS)
      IF (CHECK.GT.DIFF) DIFF=CHECK
65    CONTINUE
      SKTEST=DIFF/NX
70    RETURN
 75   J=K-1
      TEM(K)=NX-T(J)
      T(K)=T(J)+TEM(K)
      GO TO 45
 80   WRITE (6,81) (H(J),CHI(J),TEM(J),T(J),J=1,INT)
 81   FORMAT (3X,4F18.10/)
      STOP 3333
      END
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          SUBROUTINE COMPUT (ICDO,W,Y,NZ)
C
C    COMPUTE SUBROUTINE PROVIDES COMPUTATION OF PROBABILITIES FOR SELECTED
C    QUANTILE VALUES
C
          COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
          COMMON X(100),X1(3900),X2(3900),X3(3900),D(100),EMP(100),AJJ
          COMMON P1(100),P2(100),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
          COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
          COMMON LIMIT,IFACT
          DIMENSION UU(100),W(1),Y(1)
          IMPLICIT REAL*8 (A-H,O-Z)
          REAL*4 X,X1,X2,X3,D,AJJ,P1,P2,P3,P4,P5,P6,P,PL,W,Y
          REAL*4 EMP,H
          REAL*4 CHI
          IF (NX.LT.6) GO TO 200
          EN=NX
          ENN=NNX
          TRACE= (ENN-EN)/ENN
C   IF  ICDO=2 OR ICDO=99 NOT COMPUTE PARAMETERS
          IF  (ICDO.EQ.2.OR.ICDO.EQ.99) GO TO 68
          IF (NX.EQ.0) GO TO 200
          XBAR=SX/EN
          D=4.*(DLOG(XBAR)-SLX/EN)
          GAMMA=(1.+SQRT(1.+D/3.))/D
          BETA=XBAR/GAMMA
C COMPUTE  GAMMA OF GAMMA
 68       CALL GAMIT (2)
          SEE=SQRT(GAMMA)
          YOU=1./(BETA*SEE)
          DO 70 K=1,NZ
 70       UU(K)=W(K)*YOU
          DO 90 M=1,NZ
C IF PROB EXCEEDS ALLOWABLE RANGE OF THE MEAN TERMINATE COMPUTATION FOR THIS LEV
          IF (W(M).GT.(LIMIT*XBAR)) GO TO 300
          SEA=UU(M)*SEE
          Z=2.
          TERM=SEA/(PEA+Z)
          SERIES=TERM+1.
          DO 75  L=1,5000
          Z=Z+1.
          TERM=(TERM*SEA)/(PEA+Z)
          SERIES=SERIES+TERM
          IF (TERM.LT..1E-7) GO TO 80
 75       CONTINUE
C   IF FLAG=1 GAMMA>50 AND GAMMA OF GAMMA (GAM) IS COMPUTED AS LOG(GAMMA)
 80       IF  (FLAG.EQ.1.)  GO TO 81
          EYE= ((SEA**GAMMA)*EXP(-SEA)*SERIES)/GAM
          GO TO 82
 81       EYE=  EXP(LOG(SEA)*GAMMA-SEA+LOG(SERIES)-GAM)
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 82       Y(M)=1.-(((1.-TRACE)*EYE)+TRACE)
          IF ( Y(M).LE.1.) GO TO 85
           Y(M)=1.
 85       IF ( Y(M).LT..00004)  Y(M)=0.
          IF (ICDO.EQ.99) Y(M)=EYE
 90       CONTINUE
          CALL GAMIT(1)
          IF (ICDO.EQ.99) GO TO 200
          IF (ICDO.EQ.2) GO TO 99
C    COMPUTE AND ACCUMULATE EMPIRICAL PROBABILITIES
          L=1
          DEN= V -2.*C+1.
          DO 95 I=1,K1
          IF (D(I)-0.) 96,96,97
 96       EMP(I)=0.
          P1(I)=0.
          P2(I)=0.
          GO TO 95
 97       IF (D(I).GE.AJJ) GO TO 96
          EMP(I)=(L-C)/DEN
          L=L+1
 95       CONTINUE
C COMPUTE PRECIP AMOUNTS FOR COMPUTED EMPIRICAL PROBALITIES WITH BETA=1.
C
          CALL INVGAM (1.,EMP,P1,NNX,K1,K1)
C COMPUTE PRECIP AMOUNTS FOR SELECTED PROBABILITY LEVELS WITH BETA=1.
 99       CALL INVGAM (1.,PL,P3,N,NX,NNX)
          DO 100 I=1,NZ
          P4(I)=P3(I)*BETA
 100      CONTINUE
          DO 101 I=1,NNX
          P2(I)=P1(I)*BETA
 101      CONTINUE
 200      RETURN
 300      Y(M)=0.
          GO TO 90
          END
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      SUBROUTINE INVGAM(RET,PP,PK,NN,NY,NNY)
C
C  INVGAM SUBROUTINE COMPUTES QUANTILES FOR SELECTED PROBABILITIES
C
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
      COMMON X(100),X1(3900),X2(3900),X3(3900),D(100),EMP(100),AJJ
      COMMON P1(100),P2(100),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,W
      COMMON LIMIT,IFACT
      DIMENSION PP(1),PK(1)
      DIMENSION AA(11),BB(11),CC(11)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 X,X1,X2,X3,D,AJJ,P1,P2,P3,P4,P5,P6,P,PL,PP,PK
      REAL*4 EMP,H
      REAL*4 CHI
      DIMENSION TAB(52,32),PE(52),GG(45),C(6)
      DIMENSION XT1(5,45),YT1(5,20),XT2(5,45),YT2(5,85),P8(45)
      DATA YT2/     .18999065E+02,.20738106E+01,-.68135165E-01,
     1.12961368E-02,.23119701E-04,.18976302E+02,.19603549E+01,
     2-.56305560E-01,.81384021E-03,.30108261E-05,.18954790E+02,
     3.18532210E+01,-.45935583E-01,.44497553E-03,-.34198469E-05,
     4.18934417E+02,.17518204E+01,-.36819058E-01,.15884004E-03,
     5-.61299187E-05,.18915083E+02,.16556487E+01,-.28793880E-01,
     6-.60524384E-04,-.87819327E-05,.18896707E+02,.15642635E+01,
     7-.21722285E-01,-.22491058E-03,-.98675034E-05,.18879213E+02,
     8.14772703E+01,-.15488154E-01,-.34599073E-03,-.85963403E-05,
     9.18862533E+02,.13943215E+01,-.99916449E-02,-.43111618E-03,
     1-.83794802E-05,.18846607E+02,.13151072E+01,-.51463074E-02,
     2-.48854118E-03,-.71589694E-05,.18831379E+02,.12393493E+01,
     3-.87901114E-03,-.52244011E-03,-.56330222E-05,.18816802E+02,
     4.11667981E+01,.28743149E-02,-.53635992E-03,-.44050717E-05,
     5.18802828E+02,.10972293E+01,.61683989E-02,-.53802247E-03,
     6-.26424267E-05,.18789419E+02,.10304399E+01,.90522697E-02,
     7-.52506773E-03,-.13134717E-05,.18776538E+02,.96624544E+00,
     8.11568022E-01,-.50510574E-03,-.31182231E-06,.18764149E+02,
     9.90447985E+00,.13754334E-01,-.47689888E-03,.12343753E-05,
     1.18752225E+02,.84499013E+00,.15642001E-01,-.44351639E-03,
     2.21282077E-05,.18740734E+02,.78763605E+00,.17261591E-01,
     3-.40543123E-03,.30406620E-05,.18729652E+02,.73229297E+00,
     4.18638905E-01,-.36484835E-03,.42741631E-05,.18718956E+02,
     5.67884201E+00,.19796052E-01,-.32250204E-03,.46914046E-05,
     6.18708622E+02,.62717613E+00,.20754952E-01,-.27889388E-03,
     7.47532576E-05,.18698633E+02,.57719594E+00,.21533044E-01,
     8-.23499671E-03,.49439462E-05,.18688967E+02,.52880999E+00,
     9.22146657E-01,-.19137359E-03,.53347816E-05,.18679609E+02,
     1.48193506E+00,.22610711E-01,-.14838210E-03,.59856252E-05,
     2.18670541E+02,.43649124E+00,.22939552E-01,-.10596038E-03,
     3.54228773E-05,.18661750E+02,.39240622E+00,.23143595E-01,
     4-.64496935E-04,.58701783E-05,.18653221E+02,.34961189E+00,
     5.23234207E-01,-.24867369E-04,.55572522E-05,.18644942E+02,
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     8.50804721E-05,.18629084E+02,.22836946E+00,.22920166E-01,
     9.83448234E-04,.52830352E-05,.18621483E+02,.19015123E+00,
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     3.18606891E+02,.11671922E+00,.21888698E-01,.17364570E-03,
     4.33092842E-05,.18599883E+02,.81416478E-01,.21415307E-01,
     5.19959654E-03,.31207246E-05,.18593053E+02,.47002147E-01,
     6.20886193E-01,.22305861E-03,.26915513E-05,.18586395E+02,
     7.13439745E-01,.20305866E-01,.24510594E-03,.21583469E-05,
     8.18579903E+02,-.19306074E-01,.19679166E-01,.26397330E-03,
     9.20470787E-05,.18573570E+02,-.51268898E-01,.19008241E-01,
     1.28165364E-03,.92244514E-06,.18567387E+02,-.82479030E-01,
     2.18298787E-01,.29673328E-03,.10171327E-05,.18561351E+02,
     3-.11296662E+00,.17552489E-01,.30994707E-03,.77998907E-07,
     4.18555455E+02,-.14276071E+00,.16773661E-01,.32107044E-03,
     5.18750778E-06,.18549693E+02,-.17188550E+00,.15964756E-01,
     6.32965615E-03,-.62910041E-06,.18544061E+02,-.20036771E+00,
     7.15128473E-01,.33733145E-03,-.97494543E-06,.18538555E+02,
     8-.22823018E+00,.14266167E-01,.34276756E-03,-.10595267E-05,
     9.18533168E+02,-.25549580E+00,.13380836E-01,.34559443E-03,
     1-.19841424E-05,.18527897E+02,-.28218563E+00,.12474427E-01,
     2.34778615E-03,-.15192654E-05,.18522738E+02,-.30831967E+00,
     3.11549105E-01,.34757733E-03,-.29958722E-05,.18517685E+02,
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     5.18512738E+02,-.35899796E+00,.96463758E-02,.34190650E-03,
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     9-.4310352E+00,.66889109E-02,.32281659E-03,-.42123373E-05,
     1.18493919E+02,-.45447919E+00,.56805725E-02,.31308538E-03,
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      4-.26739235E-05,.18427762E+02,-.79111959E+00,-.12257598E-01,
      5-.32291580E-04,-.19806291E-05,.18424429E+02,-.80811624E+00,
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      7-.82485279E+00,-.14392048E-01,-.90644380E-04,-.12646606E-05,
      8.18417918E+02,-.84133630E+00,-.15456397E-01,-.12111576E-03,
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      1-.15173021E-03,-.13418695E-06,.18411602E+02,-.87356857E+00,
      2-.17581398E-01,-.18339908E-03,-.12741381E-06,.18408515E+02,
      3-.88932830E+00,-.18640463E-01,-.21609588E-03,.87816205E-06,
      4.18405473E+02,-.90485979E+00,-.19696706E-01,-.24856010E-03,
      5.13526246E-05,.18402476E+02,-.9201671E+00,-.20750753E-01,
      6-.28200894E-03,.14553698E-05,.18399521E+02,-.93525639E+00,
      7-.21801719E-01,-.31613981E-03,.31536027E-05,.18396610E+02,
      8-.95013255E+00,-.22849586E-01,-.35031696E-03,.37884509E-05,
      9.18393739E+02,-.96480107E+00,-.23893964E-01,-.38577629E-03,
      1.40386764E-05,.18390908E+02,-.97926653E+00,-.24936449E-01,
      2-.42111498E-03,.52538316E-05,.18388116E+02,-.99353275E+00,
      3-.25974433E-01,-.45741560E-03,.60808736E-05,.18385363E+02,
      4-.10076050E+01,-.27008920E-01,-.49447153E-03,.66241490E-05,
      5.18382647E+02,-.10214894E+01,-.28039981E-01,-.53056210E-03,
      6.78755355E-05,.18379969E+02,-.10351866E+01,-.29067934E-01,
      7-.56765876E-03,.88249211E-05/
       DATA XT2/.99881322E-01,-.18871401E+00,-.26298174E+00,-.35992913E+0
      10,.41539210E+00,.10188944E+00,-.18670548E+00,-.22319488E+00,
      2-.24591418E+00,.19433831E+00,.10316677E+00,-.18521279E+00,
      3-.19857915E+00,-.18144427E+00,.81915188E-01,.10412579E+00,
      4-.18398494E+00,-.18046147E+00,-.13668371E+00,.14835979E-01,
      5.10490290E+00,-.18292388E+00,-.16601550E+00,-.10290109E+00,
      6-.23182667E-01,.10556114E+00,-.18197942E+00,-.15395222E+00,
      7-.75935302E-01,-.64539807E-01,.10613511E+00,-.18112207E+00,
      8-.14355844E+00,-.53765694E-01,-.79489226E-01,.10664597E+00,
      9-.18033284E+00,-.13441132E+00,-.34582243E-01,-.94342556E-01,
      1.10710766E+00,-.17959847E+00,-.12622735E+00,-.18128889E-01,
      2-.11831225E+00,.10752986E+00,-.17890961E+00,-.11881266E+00,
      3-.37047166E-02,-.13076010E+00,.11055327E+00,-.17350259E+00,
      4-.67729170E-01,.83634919E-01,-.16714673E+00,.11255811E+00,
      5-.16947210E+00,-.35862646E-01,.12771319E+00,-.17481535E+00,
      6.11411220E+00,-.16611222E+00,-.12295114E-01,.15459019E+00,
      7-.14081216E+00,.11540613E+00,-.16316170E+00,.65526337E-02,
      8.17268325E+00,-.13942427E+00,.11652894E+00,-.16049117E+00,
      9.22328486E-01,.18548354E+00,-.11368517E+00,.11752992E+00,
      1-.15802537E+00,.35937668E-01,.19440329E+00,-.94569277E-01,
      2.11843945E+00,-.15571665E+00,.47921827E-01,.20074787E+00,
      3-.66326419E-01,.11927763E+00,-.15353221E+00,.58646772E-01,
      4.20535825E+00,-.41079595E-01,.12005855E+00,-.15144863E+00,
      5.68354943E-01,.20822792E+00,-.28316703E-01,.12615097E+00,
      6-.13365297E+00,.13470538E+00,.19972784E+00,.13768539E+00,
      7.13087289E+00,-.11809116E+00,.17447321E+00,.16320573E+00,
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      8.20838211E+00,.13513473E+00,-.10282062E+00,.20155322E+00,
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      1.22003818E+00,.63236123E-01,.28518445E+00,.14367840E+00,
      2-.69029590E-01,.23099400E+00,.63676307E-02,.25194664E+00,
      3.14856029E+00,-.48005425E-01,.23340690E+00,-.53432157E-01,
      4.20312472E+00,.15454652E+00,-.20720647E-01,.22289000E+00,
      5-.11362826E+00,.11900741E+00,.16331914E+00,.21881934E-01,
      6.18280759E+00,-.16324941E+00,-.23193232E-01,.16453983E+00,
      7.28029184E-01,.17509468E+00,-.16586982E+00,-.31781405E-01,
      8.16587637E+00,.34816482E-01,.16608478E+00,-.16752668E+00,
      9-.44200475E-01,.16735834E+00,.42409042E-01,.15542836E+00,
      1-.16812189E+00,-.79985776E-01,.16902856E+00,.51047592E-01,
      2.14260015E+00,-.16635372E+00,-.95712586E-01,.17095256E+00,
      3.61101655E-01,.12677782E+00,-.16220577E+00,-.11388992E+00,
      4.17323855E+00,.73184213E-01,.10658267E+00,-.15361155E+00,
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      6-.13762502E+00,-.13744437E+00,.17993202E+00,.10935379E+00,
      7.39336341E-01,-.10749182E+00,-.14380852E+00,.18613103E+00,
      8.14379800E+00,-.32746960E-01,-.38126106E-01,-.99810101E-01,
      9.18703746E+00,.14890406E+00,-.44009373E-01,-.26291367E-01,
      1-.11335289E+00,.18804071E+00,.15457542E+00,-.56686305E-01,
      2-.11950671E-01,-.92908377E-01,.18916572E+00,.16095924E+00,
      3-.71149663E-01,.45311859E-02,-.72725656E-01,.19044878E+00,
      4.16827035E+00,-.87967974E-01,.24175104E-01,-.48737463E-01,
      5.19194563E+00,.17684048E+00,-.10801073E+00,.48035983E-01,
      6-.25956831E-01,.19374878E+00,.18722062E+00,-.13273934E+00,
      7.78644621E-01,-.60990423E-02,.19602958E+00,.20043641E+00,
      8-.16491819E+00,.11992006E+00,.61549507E-01,.19916699E+00,
      9.21876769E+00,-.21077093E+00,.18107879E+00,.14878758E+00,
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      2.34977127E+00/
       DATA YT1/       .20552448E+02,-.30696131E+01,-.26695220E+00,
      1.60260049E-02,-.73553285E-04,.20432134E+02,-.24206541E+01,
      2-.76621535E-01,-.34313134E-02,.12894464E-03,.20290268E+02,
      3-.18825336E+01,.24652504E-01,-.49226591E-02,.67347837E-04,
      4.20151358E+02,-.14298066E+01,.78158101E-01,-.38931192E-02,
      5-.14304087E-04,.20022172E+02,-.10427771E+01,.10398814E+00,
      6-.22055723E-02,-.63564203E-04,.19904029E+02,-.70710676E+00,
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      8-.41233718E+00,.11117399E+00,.83419816E-03,-.70415776E-04,
      9.19698510E+02,-.15071553E+00,.10250214E+00,.18627997E-02,
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      3.73097500E-01,.28832504E-02,.96900150E-05,.19451746E+02,
      4.48743608E+00,.54965963E-01,.29433733E-02,.34073779E-04,
      5.19382188E+02,.66326397E+00,.35608267E-01,.27523257E-02,
      6.52516051E-04,.19317757E+02,.82490166E+00,.15518014E-01,
      7.23470276E-02,.62342092E-04,.19257874E+02,.97419274E+00,
      8-.49549665E-02,.17588355E-02,.62925241E-04,.19202042E+02,
      9.11126633E+01,-.25566356E-01,.10165045E-02,.55079382E-04,
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      1.19149829E+02,.12415842E+01,-.46139671E-01,.14464919E-03,
      2.37900331E-04,.19100868E+02,.13620240E+01,-.66549905E-01,
      3-.83644394E-03,.14287078E-04,.19054831E+02,.14748976E+01,
      4-.86703487E-01,-.19158115E-02,-.90430397E-05,.19011456E+02,
      5.15B09643E+01,-.10655572E+00,-.30623688E-02,-.50308501E-04,
      6.18970486E+02,.16809039E01,-.12605538E+00,-.42717591E-02,
      7-.98798473E-04/
       DATA XT1/.68440903E-01,.13539709E+00,-.22276772E+00,-.32197761E+00
      1,-.13366428E+00,.70154770E-01,.14121096E+00,-.20739324E+00,
      2-.24387948E+00,-.90781245E-01,.71286686E-01,.14473000E+00,
      3-.19696011E+00,-.19828904E+00,-.10239516E+00,.72157844E-01,
      4.14727630E+00,-.18882653E+00,-.16592193E+00,-.10873253E+00,
      5.72877158E-01,.14927665E+00,-.18207035E+00,-.14089894E+00,
      6-.98523796E-01,.73495800E-01,.15092637E+00,-.17624132E+00,
      7-.12043555E+00,-.82914629E-01,.74042243E-01,.15233050E+00,
      8-.17107946E+00,-.10315707E+00,-.69598948E-01,.74534106E-01,
      9.15355322E+00,-.16642859E+00,-.88146949E-01,-.59247622E-01,
      1.74983064E-01,.15463582E+00,-.16218118E+00,-.74920111E-01,
      2-.45638966E-01,.75397306E-01,.15560697E+00,-.15826359E+00,
      3-.63128816E-01,-.33664710E-01,.78466676E-01,.16201847E+00,
      4-.12934751E+00,.14208264E-01,.52010895E-01,.80601493E-01,
      5.16572263E+00,-.10949622E+00,.59031710E-01,.10588637E+00,
      6.82310846E-01,.16827926E+00,-.93831376E-01,.90242719E-01,
      7.14087334E+00,.83770337E-01,.17019135E+00,-.80647535E-01,
      8.11409116E+00,.16383555E+00,.85063543E-01,.17168716E+00,
      9-.69128131E-01,.13310291E+00,.18165051E+00,.86237339E-01,
      1.17288987E+00,-.58815663E-01,.14875102E+00,.18987157E+00,
      2.87320986E-01,.17387421E+00,-.49421235E-01,.16193758E+00,
      3.19637927E+00,.88334038E-01,.17468793E+00,-.40751650E-01,
      4.17320441E+00,.19992577E+00,.89290312E-01,.17536452E+00,
      5-.32673356E-01,.18294635E+00,-.19853831E+00,.97161781E-01,
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      8.33448469E-02,.11009157E+00,.17164814E+00,.11392607E+00,
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      1.14736432E+00,.19684960E+00,-.22394455E+00,.12382559E+00,
      2.15391561E+00,.17676890E+00,.14950527E+00,-.29816413E+00,
      3.13228269E+00,.13830843E+00,.20150917E+00,.84872764E-01,
      4-.32660397E+00,.14323056E+00,.11364561E+00,.21815568E+00,
      5.57899927E-04,-.27950802E+00,.16034850E+00,.66692577E-01,
      6.21173993E+00,-.10574522E+00,-.98492325E-01,.16282487E+00,
      7.59175105E-01,.20779170E+00,-.11667903E+00,-.67457137E-01,
      8.16556126E+00,.50678327E-01,.20259932E+00,-.12736135E+00,
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      1-.13738096E+00,-.45268249E-02,.17211527E+00,.2956268E-01,
      2.18677698E+00,-.14630390E+00,.35312979E-01,.17618246E+00,
      3.15951663E-01,.17467069E+00,-.15341020E+00,.75727046E-01,
      4.18107789E+00,-.90786713E-03,.15791095E+00,-.15695878E+00,
      5.11480029E+00,.18726705E+00,-.22914627E-01,.13350043E+00,
      6-.15382489E+00,.15296481E+00,.19578077E+00,-.54334491E-01,
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      7.94509002E-01,-.13618683E+00,.17322981E+00,.20984487E+00,
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      1.14537149E+00,.21425678E+00,-.12644214E+00,-.88664720E-02,
      2-.49088761E-01,.12503749E+00,.21687243E+00,-.13702993E+00,
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      4-.14926875E+00,-.44702873E-01,-.10820995E-01,.88586113E-01,
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      6.56194542E-01,.22764995E+00,-.18152426E+00,-.97348141E-01,
      7.50052843E-01,.18868993E-01,.23308347E+00,-.20444935E+00,
      8-.13602611E+00,.97684746E-01,-.44241221E-01,.24063008E+00,
      9-.23679031E+00,-.19212919E+00,.16944486E+00,-.12515575E+00,
      1.25325921E+00,-.29211561E+00,-.29178592E+00,.30643882E+00,
      2-.36690261E+00/
       DATA PB/.001,.002,.003,.004,.005,.006,.007,.008,.009,.01,.02,.03,
      1.04,.05,.06,.07,.08,.09, .1,.2,.3,.4,.5,.6,.7,.8,.9,.91,.92,.93,
      2.94,.95,.96,.97,.98,.99,.991,.992,.993,.994,.995,.996,.997,.998,
      3.999/
       DATA AA/.935,.952,.969,.967,.950,.986,.994,.971,.957,.937,.937/
       DATA BB/-1.2,-.8,-.5,-.4,.0,.2,.4,1.2,1.8,2.2,2.2/
       DATA CC/.05,.1,.2,.25,.4,.5,.6,.8,.9,.95,.99/
C VALUES FOR G=.01
       DATA TAB/45*1.6658,1.6656,1.6643,1.6615,1.6522,1.6388,1.6277,
      *1.6040,
C VALUES FOR G=.1
      $             12*1.6084,20*1.6083,3*1.6082,1.6081,1.6080,1.6079,
      11.6076,1.6069,1.6057,1.6033,1.5987,1.5973,1.5937,1.5884,1.5799,
      21.5732,1.5619,1.5514,1.5443,1.5309,
C VALUES FOR G=.2
      31.5514,29*1.5515,1.5514,1.5513,1.5512,1.5510,1.5507,1.5502,1.5499,
      41.5496,1.5487,1.5473,1.5454,1.5426,1.5382,1.5370,1.5341,1.5301,
      51.5242,1.5196,1.5123,1.5057,1.5012,1.4928,
C VALUES FOR G=.3
      61.5009,2*1.5010,25*1.5009,2*1.5008,1.5006,1.5005,1.5002,1.4999,
      71.4995,2*1.4990,1.4984,1.4975,1.4964,1.4949,1.4929,1.4900,1.4892,
      81.4874,1.4849,1.4813,1.4786,1.4743,1.4703,1.4678,1.4629,
C VALLUES FOR G=.4
      928*1.4554,1.4553,1.4552,1.4551,1.4550,1.4548,1.4546,1.4543,1.4540,
      A1.4538,1.4536,1.4532,1.4526,1.4518,1.4509,1.4494,1.4491,1.4482,
      81.4471,1.4455,1.4442,1.4423,1.4405,1.4394,1.4372,
C VALUES FOR G=.5
      C52*1.4142,
C VALUES FOR G=.6
      D20*1.3766,6*1.3767,1.3768,1.3769,1.3770,1.3771,1.3773,1.3775,
      E1.3778,1.3780,1.3783,1.3787,1.3789,1.3791,1.3796,1.3801,1.3808,
      F1.3817,1.3830,1.3833,1.3840,1.3850,1.3864,1.3874,1.3891,1.3905,
      G1.3915,1.3933,
C VALUES FOR G=.7
      H20*1.3422,5*1.3423,1.3424,1.3426,1.3428,1.3431,1.3435,1.3439,
      I1.3443,1.3448,1.3453,1.3459,1.3467,1.3471,1.3475,1.3484,1.3495,
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     J1.3508,1.3525,1.3549,1.3555,1.3569,1.3587,1.3613,1.3632,1.3662,
     K1.3689,1.3707,1.3742,
C VALUES FOR G=.8
     L10*1.3103,5*1.3104,3*1.3105,2*1.3106,3*1.3107,3*1.3108,1.3112,
     M1.3117,1.3122,1.3127,1.3134,1.3140,1.3148,1.3156,1.3165,1.3176,
     N1.3183,1.3187,1.3201,1.3217,1.3236,1.3260,1.3293,1.3302,1.3321,
     O1.3347,1.3383,1.3410,1.3452,1.3490,1.3516,1.3565,
C VALUES FOR G=.9
     P1.2807,7*1.2808,2*1.2809,2*1.2810,2*1.2811,2*1.2812,2*1.2813,
     Q2*1.2814,2*1.2815,2*1.2816,2*1.2817,1.2824,1.2830,1.2838,1.2845,
     R1.2854,1.2863,1.2873,1.2883,1.2896,1.2909,1.2919,1.2925,1.2942,
     S1.2962,1.2987,1.3018,1.3059,1.3070,1.3094,1.3126,1.3171,1.3205,
     T1.3257,1.3305,1.3338,1.3398,
C VALUES FOR G=1.0
     U1.2533,6*1.2534,2*1.2535,1.2536,1.2537,2*1.2538,1.2539,1.2540,
     V2*1.2541,1.2542,1.2543,1.2544,2*1.2545,1.2546,1.2547,1.2548,
     W1.2549,1.2557,1.2566,1.2575,1.2585,1.2596,1.2607,1.2620,1.2633,
     X1.2648,1.2664,1.2664,1.2683,1.2704,1.2728,1.2757,1.2793,1.2843,
     Y1.2855,1.2884,1.2921,1.2975,1.3014,1.3077,1.3133,1.3171,1.3243,
C VALUES FOR G=1.1
     Z1.2277,2*1.2278,5*1.2279,1.2281,1.2282,1.2283,1.2284,1.2285,
     11.2286,1.2287,1.2288,1.2289,1.2290,1.2291,1.2293,1.2294,1.2295,
     21.2296,1.2297,1.2298,1.2298,1.2310,1.2321,1.2332,1.2344,1.2357,
     31.2371,1.2386,1.2402,1.2419,1.2438,1.2451,1.2459,1.2483,1.2511,
     41.2544,1.2586,1.2642,1.2656,1.2689,1.2731,1.2792,1.2836,1.2907,
     51.2971,1.3014,1.3097,
C VALUES FOR G=1.2
     62*1.2038,1.2039,3*1.2040,2*1.2041,1.2043,1.2044,1.2046,1.2047,
     71.2049,1.2050,1.2051,1.2053,1.2054,1.2056,1.2057,1.2058,1.2060,
     81.2061,1.2062,1.2064,1.2065,1.2066,1.2079,1.2093,1.2106,1.2121,
     91.2135,1.2151,1.2168,1.2186,1.2206,1.2227,1.2241,1.2251,1.2278,
     A1.2309,1.2346,1.2392,1.2455,1.2470,1.2507,1.2553,1.2620,1.2670,
     B1.2748,1.2818,1.2866,1.2957,
C VALUES FOR G=1.3
     C1.1812,1.1814,1.1815,1.1815,2*1.1816,2*1.1817,1.1820,1.1822,
     D1.1824,1.1826,1.1827,1.1829,1.1831,1.1833,1.1834,1.1836,1.1838,
     E1.1839,1.1841,1.1842,1.1844,1.1846,1.1847,1.1849,1.1864,1.1880,
     F1.1895,1.1912,1.1928,1.1946,1.1965,1.1985,1.2007,1.2030,1.2046,
     G1.2057,1.2086,1.2120,1.2161,1.2211,1.2279,1.2296,1.2335,1.2386,
     H1.2459,1.2512,1.2597,1.2674,1.2726,1.2826,
C VALUES FOR G=1.4
     I1.1600,1.1602,1.1604,2*1.1605,1.1606,1.1606,1.1607,1.1611,1.1613,
     J1.1615,1.1617,1.1620,1.1622,1.1624,1.1626,1.1628,1.1630,1.1632,
     K1.1634,1.1636,1.1638,1.1640,1.1641,1.1643,1.1645,1.1664,1.1680,
     L1.1698,1.1716,1.1735,1.1754,1.1775,1.1797,1.1820,1.1846,1.1863,
     M1.1874,1.1906,1.1943,1.1987,1.2041,1.2114,1.2132,1.2174,1.2228,
     N1.2306,1.2363,1.2455,1.2537,1.2593,1.2700,
CVALUES FOR G=1.5
     O1.1401,1.1403,1.1405,1.1406,1.1407,1.1408,2*1.1409,1.1413,1.1416,
     P1.1419,1.1422,1.1424,1.1427,1.1429,1.1432,1.1434,1.1436,1.1439,
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     Q1.1441,1.1443,1.1445,1.1447,1.1449,1.1451,1.1454,1.1474,1.1493,
     R1.1513,1.1532,1.1553,1.1574,1.1596,1.1620,1.1645,1.1673,1.1691,
     S1.1703,1.1737,1.1776,1.1823,1.1880,1.1958,1.1977,1.2022,1.2079,
     T1.2162,1.2223,1.2320,1.2407,1.2467,1.2581,
C VALUES FOR G=1.6
     U1.1211,1.1215,1.1218,1.1219,1.1220,1.1221,2*1.1222,1.1227,1.1230,
     V1.1234,1.1237,1.1240,1.1243,1.1246,1.1248,1.1251,1.1254,1.1256,
     W1.1259,1.1261,1.1264,1.1266,1.1268,1.1271,1.1273,1.1295,1.1317,
     X1.1338,1.1359,1.1381,1.1404,1.1428,1.1453,1.1480,1.1509,1.1529,
     Y1.1541,1.1578,1.1619,1.1668,1.1729,1.1810,1.1830,1.1877,1.1938,
     Z1.2025,1.2089,1.2191,1.2283,1.2346,1.2466,
C VALUES FOR G=1.7
     11.1033,1.1037,1.1040,1.1041,1.1042,1.1044,1.1045,1.1046,1.1051,
     21.1055,1.1059,1.1062,1.1066,1.1069,1.1072,1.1075,1.1078,1.1081,
     31.1084,1.1087,1.1090,1.1092,1.1095,1.1098,1.1100,1.1103,1.1127,
     41.1150,1.1173,1.1196,1.1219,1.1243,1.1268,1.1295,1.1324,1.1355,
     51.1375,1.1389,1.1427,1.1470,1.1521,1.1585,1.1670,1.1691,1.1740,
     61.1804,1.1894,1.1961,1.2067,1.2164,1.2229,1.2355,
C VALUES FOR G=1.8
     71.0863,1.0868,1.0871,1.0873,1.0874,1.0876,1.0877,1.0878,1.0884,
     81.0889,1.0893,1.0897,1.0901,1.0905,1.0908,1.0912,1.0915,1.0918,
     91.0921,1.0924,1.0927,1.0930,1.0933,1.0936,1.0939,1.0941,1.0968,
     A1.0992,1.1017,1.1041,1.1066,1.1091,1.1118,1.1146,1.1176,1.1208,
     B1.1230,1.1244,1.1283,1.1329,1.1382,1.1448,1.1536,1.1559,1.1610,
     C1.1676,1.1770,1.1840,1.1950,1.2051,1.2119,1.2251,
C VALUES FOR G=1.9
     D1.0702,1.0707,1.0711,1.0713,1.0715,1.0716,1.0718,1.0719,1.0725,
     E1.0731,1.0736,1.0740,1.0745,1.0749,1.0752,1.0756,1.0760,1.0763,
     F1.0767,1.0770,1.0773,1.0776,1.0779,1.0782,1.0785,1.0788,1.0817,
     G1.0843,1.0869,1.0894,1.0920,1.0947,1.0975,1.1004,1.1035,1.1069,
     H1.1091,1.1106,1.1147,1.1194,1.1250,1.1318,1.1409,1.1432,1.1485,
     I1.1553,1.1651,1.1723,1.1837,1.1941,1.2012,1.2149,
C VALUES FOR G=2.0
     J1.0548,1.0555,1.0559,1.0561,1.0563,1.0565,1.0566,1.0568,1.0575,
     K1.0581,1.0586,1.0591,1.0596,1.0600,1.0604,1.0608,1.0612,1.0616,
     L1.0620,1.0623,1.0627,1.0630,1.0633,1.0637,1.0640,1.0643,1.0673,
     M1.0701,1.0728,1.0755,1.0782,1.0810,1.0839,1.0869,1.0901,1.0936,
     N1.0960,1.0975,1.1017,1.1066,1.1123,1.1194,1.1288,1.1312,1.1366,
     O1.1436,1.1537,1.1612,1.1730,1.1837,1.1911,1.2052,
C VALUES FOR G=2.1
     P1.04012,1.04093,1.04145,1.04163,1.04190,1.04210,1.04225,1.04245,
     Q1.04320,1.04385,1.04438,1.04495,1.04543,1.04588,1.04635,1.04681,
     R1.04720,1.04758,1.04806,1.04841,1.04873,1.04910,1.04946,1.04986,
     S1.05015,1.05051,1.05366,1.05655,1.05941,1.06221,1.06501,1.06792,
     T1.07092,1.07408,1.07738,1.08098,1.08343,1.08498,1.08935,1.09433,
     U1.10026,1.10750,1.11723,1.11961,1.12523,1.13245,1.14281,1.15043,
     V1.16261,1.17368,1.18122,1.19580,
C VALUES FOR G=2.2
     W1.0261,1.0270,1.0276,1.0278,1.0281,1.0283,1.0285,1.0287,1.0295,
     X1.0302,1.0308,1.0314,1.0319,1.0324,1.0329,1.0334,1.0338,1.0342,
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      Y1.0347,1.0351,1.0354,1.0358,1.0362,1.0366,1.0369,1.0373,1.0406,
      Z1.0436,1.0466,1.0495,1.0524,1.0554,1.0585,1.0618,1.0652,1.0689,
      11.0714,1.0730,1.0775,1.0826,1.0887,1.0961,1.1061,1.1085,1.1143,
      21.1217,1.1323,1.1401,1.1526,1.1640,1.1717,1.1867,
C VALUES FOR G=2.3
      31.01280,1.01373,1.01437,1.01463,1.01493,1.01512,1.01538,1.01558,
      41.01642,1.01717,1.01790,1.01848,1.01903,1.01960,1.02008,1.02058,
      51.02103,1.02150,1.02192,1.02238,1.02275,1.02313,1.02353,1.02392,
      61.02428,1.02468,1.02815,1.03131,1.03441,1.03741,1.04042,1.04345,
      71.04666,1.05007,1.05357,1.05738,1.05993,1.06158,1.06618,1.07145,
      81.07765,1.08526,1.09540,1.09791,1.10381,1.11136,1.12216,1.13025,
      91.14298,1.15465,1.16256,1.17791,
C VALUES FOR G=2.4
      A1.0000,1.0010,1.0017,1.0020,1.0023,1.0025,1.0028,1.0030,1.0039,
      B1.0047,1.0055,1.0061,1.0067,1.0073,1.0078,1.0083,1.0088,1.0093,
      C1.0097,1.0102,1.0106,1.0110,1.0114,1.0118,1.0122,1.0126,1.0162,
      D1.0195,1.0227,1.0258,1.0289,1.0320,1.0353,1.0388,1.0424,1.0463,
      E1.0489,1.0506,1.0553,1.0607,1.0670,1.0748,1.0851,1.0877,1.0937,
      F1.1014,1.1124,1.1207,1.1337,1.1456,1.1537,1.1694,
C VALUES FOR G=2.5
      G0.98771,0.98881,0.98962,0.98992,0.99021,0.99047,0.99077,0.99096,
      H0.99197,0.99282,0.99362,0.99427,0.99492,0.99552,0.99607,0.99656,
      I0.99713,0.99762,0.99808,0.99858,0.99897,0.99943,0.99982,1.00030,
      J1.00068,1.00108,1.00477,1.00817,1.01148,1.01470,1.01782,1.02111,
      K1.02445,1.02800,1.03171,1.03565,1.03830,1.04006,1.04486,1.05036,
      L1.05676,1.06472,1.07523,1.07788,1.08398,1.09185,1.10305,1.11143,
      M1.12477,1.13686,1.14511,1.16118,
C VALUES FOR G=2.6
      N0.9759,0.9771,0.9780,0.9783,0.9786,0.9789,0.9792,0.9794,0.9805,
      O0.9814,0.9822,0.9829,0.9836,0.9842,0.9848,0.9853,0.9859,0.9864,
      P0.9869,0.9874,0.9878,0.9883,0.9887,0.9892,0.9896,0.9900,0.9938,
      Q0.9973,1.0007,1.0040,1.0072,1.0106,1.0140,1.0176,1.0214,1.0254,
      R1.0281,1.0299,1.0348,1.0404,1.0469,1.0550,1.0657,1.0684,1.0746,
      S1.0826,1.0940,1.1025,1.1161,1.1284,1.1368,1.1532,
C VALUES FOR G=2.7
      T0.96460,0.96588,0.96685,0.96713,0.96750,0.96780,0.96808,0.96835,
      U0.96950,0.97045,0.97123,0.97200,0.97276,0.97335,0.97401,0.97457,
      V0.97508,0.97566,0.97615,0.97663,0.97710,0.97760,0.97806,0.97848,
      W0.97895,0.97935,0.98331,0.98691,0.99036,0.99370,0.99708,1.00046,
      X1.00397,1.00762,1.01146,1.01557,1.01832,1.02012,1.02513,1.03085,
      Y1.03745,1.04563,1.05648,1.05925,1.06555,1.07365,1.08527,1.09393,
      Z1.10766,1.12023,1.12880,1.14543,
C VALUES FOR G=2.8
      10.9537,0.9551,0.9561,0.9564,0.9568,0.9571,0.9574,0.9577,0.9589,
      20.9599,0.9607,0.9615,0.9623,0.9629,0.9636,0.9642,0.9647,0.9653,
      30.9658,0.9663,0.9668,0.9673,0.9678,0.9682,0.9687,0.9691,0.9732,
      40.9769,0.9804,0.9838,0.9873,0.9907,0.9943,0.9980,1.0019,1.0061,
      51.0089,1.0107,1.0158,1.0216,1.0283,1.0366,1.0476,1.0504,1.0568,
      61.0650,1.0768,1.0856,1.0995,1.1123,1.1210,1.1379,
C VALUES FOR G=2.9
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      70.94320,0.94473,0.94575,0.94608,0.94650,0.94680,0.94713,0.94745,
      80.94870,0.94975,0.95058,0.95140,0.95221,0.95285,0.95356,0.95417,
      90.95473,0.95531,0.95585,0.95638,0.95690,0.95740,0.95791,0.95833,
      A0.95885,0.95925,0.96346,0.96726,0.97081,0.97430,0.97783,0.98131,
      B0.98497,0.98872,0.99271,0.99697,0.99982,1.00162,1.00678,1.01264,
      C1.01945,1.02788,1.03903,1.04185,1.04835,1.05665,1.06857,1.07748,
      D1.09161,1.10458,1.11340,1.13058,
C VALUES FOR G=3.0
      E0.9331,0.9348,0.9358,0.9362,0.9366,0.9369,0.9373,0.9376,0.9389,
      F0.9400,0.9409,0.9417,0.9425,0.9432,0.9439,0.9445,0.9452,0.9457,
      G0.9463,0.9469,0.9474,0.9479,0.9484,0.9489,0.9494,0.9498,0.9541,
      H0.9580,0.9616,0.9652,0.9687,0.9723,0.9760,0.9798,0.9839,0.9882,
      I0.9911,0.9929,0.9981,1.0040,1.0109,1.0195,1.0308,1.0336,1.0402,
      J1.0486,1.0606,1.0696,1.0840,1.0971,1.1060,1.1235,
C VALUES FOR G=3.1
      K.92342,.92515,.92622,.92665,.92705,.92741,.92775,.92806,.92942,
      L.93043,.93151,.93238,.93318,.93392,.93462,.93528,.93590,.93651,
      M.93708,.93764,.93818,.93871,.93921,.93971,.94020,.94068,.94505,
      N.94898,.95270,.95632,.95992,.96356,.96730,.97119,.97529,.97967,
      O.98257,.98443,.98971,.99569,1.00269,1.01130,1.02273,1.02558,
      P1.03222,1.04074,1.05293,1.06204,1.07653,1.08980,1.09886,1.11655/
C TABLE OF PROBABILITIES
      DATA  PE/.001,.003,.005,.006,.007,.008,.009,.010,.015,.020,.025,
     1.030,.035,.040,.045,.050,.055,.060,.065,.070,.075,.080,.085,.090,
     2.095,.100,.150,.200,.250,.300,.350,.400,.450,.500,.550,.600,.631,
     3.650,.700,.750,.800,.850,.900,.910,.930,.950,.970,.980,.990,.995,
     4.997,.999/
      DATA C/2.515517,.802853,.010328,1.432788,.189269,.001308/
C TABLE OF GAMMA VALUES
      DATA  GG/.01,.1,.2,.3,.4,.5,.6,.7,.8,.9,1.0,1.1,1.2,1.3,1.4,1.5,
     11.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,
     23.2,3.3,3.4,3.5,3.6,3.7,3.8,3.9,4.0,4.1,4.2,4.3,4.4/
C DETERMINANT SOLUTION STATEMENT
      DET(A11,A22,A33,A12,A21,A13,A31,A23,A32)= A11*(A22*A33-A32*A23)
     1                                         +A12*(A31*A23-A21*A33)
     2                                         +A13*(A21*A32-A31*A22)
      PIE=SQRT(6.28318531)
      XD=0.
      IJJ=0
      SG=SQRT(GAMMA)
      IF (PEA.GE..1) SP=SQRT(PEA)
      EN=NY
      ENN=NNY
      Q= (ENN-EN)/ENN
      DO  55  I=1,NN
      IJ=0
      PK(I)=0.
      OXD=10.
      CHECK=1.E-4
C  OL IS THE ADJUSTED PROB LEVEL TAKING INTO ACCOUNT THE ZERO PORTION OF THE
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C  DISTRIBUTION
      DL=(PP(I)-Q)/(1.-Q)
      IF (DL.LT..001 ) GO TO 55
      IF (DL.GE.1.) GO TO 7
      IF (GAMMA.GE.3.0) GO TO 18
C FIND THE ROW NUMBER OF THE H TABLE FROM THE PROBABILITY VALUE
      DO 5 K=1,52
      IF (DL.LE.PE(K)) GO TO 8
 5    CONTINUE
      WRITE (6,6) I,K,DL,GAMMA,BET,PP(I)
 6    FORMAT (1X,'I = ',I4,'K= ',I4,' DL = ',F9.5,' GAMMA = ',F9.3,
     1 ' BET = ',F9.3,' PP(I)= ',F9.3)
      STOP 4444
 7    WRITE (6,9) BET,PP(I),NN,NY,NNY,I,DL,Q
 9    FORMAT (1X,2E15.9,4I7,2E15.9)
      GO TO 55
C FIND THE COLUMN NUMBER OF THE H TABLE FROM THE GAMMA VALUE
 8    DO 10 J=1,44
      IF (GAMMA.LE.GG(J)) GO TO 12
 10   CONTINUE
      WRITE (6,11) I,J,DL,GAMMA,BET,PP(I)
 11   FORMAT (1X,' I = ',I4, ' J= ',I4,' DL = ',F9.5,' GAMMA = ',F9.3,
     1' BET = ',F9.3,' PP(I)= ',F9.3)
      STOP 4444
C FIND THE CONSTANT VALUES FOR THE DETERMINANT SOLUTION
 12   B=1.
      IF ((PE(K)-DL).GT.(DL-PE(K-1)).AND.K.GT.1) K=K-1
      IF (J.NE.1) J=J-1
      GM1=GG(J)                                                      A12
      GP0=GG(J+1)                                                    A22
      GP1=GG(J+2)                                                    A32
      S1= GM1*GM1                                                    A11
      S2= GP0*GP0                                                    A21
      S3= GP1*GP1                                                    A31
      V1= TAB(K,J)
      V2= TAB(K,J+1)
      V3= TAB(K,J+2)
      D =DET(S1,GP0,B,GM1,S2,B,S3,B,GP1)
      D1=DET(V1,GP0,B,GM1,V2,B,V3,B,GP1)/D
      D2=DET(S1,V2,B,V1,S2,B,S3,B,V3)/D
      D3=DET(S1,GP0,V3,GM1,S2,V1,S3,V2,GP1)/D
C
C
      HOFG=D1*GAMMA*GAMMA+D2*GAMMA+D3
C
      IF (GAMMA.LT.3.0) GO TO 60
 18   SMLG=0.
      SMEX=0.
      ALG= LOG(GAMMA)
      G2=GAMMA*GAMMA
```



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      G3=G2*GAMMA
      G5=G3*G2
      G7=G5*G2
      G9=G7*G2
      G11=G9*G2
      G13=G11*G2
      G15=G13*G2
      ALGG=(GAMMA-.5)*ALG-GAMMA+.5* LOG(6.2831851)+1./(12*GAMMA)-
     11./(360.*G3)+1./(1260.*G5)-1./(1680.*G7)+5./(5940.*G9)-691./
     2(360360.*G11)+7./(1092.*G13)-3617./(122400.*G15)
      DO 19 J=1,45
      IF (DL.LE.PB(J)) GO TO 20
 19   CONTINUE
      STOP
 20   TAUL= EXP((LOG(DL)+ALGG+ALG)/GAMMA)
      IF (GAMMA.GT.20) GO TO 30
      M=GAMMA-2
      RR=XT1(1,J)*YT1(1,M)+XT1(2,J)*YT1(2,M)+XT1(3,J)*YT1(3,M)
     1+XT1(4,J)*YT1(4,M)
 25   T=TAUL*RR
      DO 26 K=1,5000
      SMLG=0.
      SMEX=0.
      ALDGT= LOG(T)
      DO 23 J=1,5000
      SMLG=SMLG+ LOG(GAMMA+J)
      EX= EXP(J*ALOGT-SMLG)
      EXX=ABS(EX)
      IF (EXX.LE..000001) GO TO 241
      SMEX=SMEX+EX
 23   CONTINUE
 241  FOFT= EXP(GAMMA*ALOGT-T-ALGG-ALG)*(1.+SMEX)
      AL=ABS(FOFT-DL )
      IF (AL.LT.1.E-9)  GO TO 27
      AK=1.
      IF (FOFT.GT.DL ) AK=-AK
      IF (FOFT.EQ.DL ) GO TO 27
      TT=T+AK*EXP(  LOG(AL)+T-PEA*ALOGT+ALGG)
      T=TT
 26   CONTINUE
      STOP 5555
 27   PK(I)=T*BET
 55   CONTINUE
      RETURN
 30   M=GAMMA-19
      RR=XT2(1,J)*YT2(1,M)+XT2(2,J)*YT2(2,M)+XT2(3,J)*YT2(3,M)
     1+XT2(4,J)*YT2(4,M)+XT2(5,J)*YT2(5,M)
      GO TO 25
C  MAKE APPROXIMATION FOR SMALL GAMMA
 60   PN1OFX=(1.+(DL)**(1./(GAMMA+GAMMA)))/2.
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FORTRAN IVL27 SOURCE PROGRAM  INVGAM   SUBROUTINE              03/25/77   PAGE 0025

      R= (DLOG(1./((1.-PN1OFX)*(1.-PN1OFX))))**.5
      R2= R*R
      X0= R- (C(1)+C(2)*R+C(3)*R2)/(1.+C(4)*R+C(5)*R2+C(6)*R2*R)
C
C
 21   XX=X0
      XTWO=XX*XX
      TERM=XX
      SERIES=XX
      ZZ=1.
      Z=1.
      DO 24  L=1,5000
      TERM=XTWO*TERM
      Z=Z+2.
      ZZ=ZZ*Z
      T=TERM/ZZ
      IF (T.LT.1.E- 6) GO TO 266
      SERIES=SERIES+T
 24   CONTINUE
      STOP 5555
C
 266  FPRIME= EXP(-XTWO*.5)/PIE
      FOFX= .5 + FPRIME*SERIES-PN1OFX
      X0= XX- FOFX/FPRIME
      DIF= ABS (X0-XX)
      IF (DIF.GT..0001) GO TO 21
      PK(I)=((X0/HOFG)**2)*BET
      GO TO 55
      END
```

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FORTRAN IVL27 SOURCE PROGRAM  PRINT    SUBROUTINE              03/25/77   PAGE 0026

      SUBROUTINE PRINT (Y,I,M,ICOD)
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
      COMMON X(100),X1(3900),X2(3900),X3(3900),D(100),EMP(100),AJJ
      COMMON P1(100),P2(100),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
      COMMON LIMIT,IFACT
      DIMENSION Y(1)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION KP(52)
      REAL*4 X,X1,X2,X3,D,AJJ,P1,P2,P3,P4,P5,P6,P,PL,Y,Z(75)
      REAL*4 EMP,H
      REAL*4 CHI
      IF (NX.LT.6) GO TO 26
      EN=NX
      ENN=NNX
      TRACE=(ENN-EN)/ENN*100.
      AI=I
      AJ=M
      ANX=NX
      ANNX=NNX
      ANUM=NUM
      ACOD=ICOD
      LINE=52
      IF (ICOD.EQ.2)  GO TO  50
      V=0.
      L=1
      K=I*NUM-(NUM-1)
      NN=K+NUM-1
      IF  (N.GT.NUM) NN=NN+N-NUM
      DO 10 J=1,NUM
      Z(J)=0.
      IF (D(J).GE.AJJ) GO TO 10
      Z(J)=D(J)/BETA
 10   CONTINUE
      WRITE (7,69) ID1,AI,AJ,ANX,ANNX,XBAR,ALPHA,BETA,GAMMA,QQ,PROB,
     1SKTEST,ANUM,ACOD
 69   FORMAT (I5,13F11.3)
      DO 25 J=K,NN
      IF (LINE.LT.45) GO TO 6
      LINE=1
      WRITE (6,1) ASTN
 1    FORMAT (1H1,33X,8A8//)
      WRITE (6,2)
 2    FORMAT (7X,'STATION     I        J      NX      NNX       XBAR        A
     1LPHA       BETA       GAMMA        X2        PROB       K-S')
      WRITE (6,3) ID1,I,M,NX,NNX,XBAR,ALPHA,BETA,GAMMA,QQ,PROB,SKTEST
 3    FORMAT (6X,3I7,2I8,5F11.3,2F10.3//)
      WRITE (6,4)
 4    FORMAT (53X,'EMP PROB  EMP PROB  SELECTED  SELECTED  SELECTED
     1GRAPH   SELECTED   EXC PRB'/16X,'ENTRY     ORDER      DATA       E
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FORTRAN IVL27 SOURCE PROGRAM  PRINT     SUBROUTINE             03/25/77    PAGE 0027

          2MP  QUANTILE  QUANTILE       PROB  QUANTILE  QUANTILE      PROB    Q
          3UANTITY      FOR'/ 8X,  'SEQ      DATA       DATA       /BETA       PRO
          48       B=1    B=BETA     VALUES         B=1    B=BETA      (X>0)
          5LEVELS   PCP LVL'/)
     6    IF (LINE.GT.N.OR.V.EQ.1.) GO TO 30
          IF  (LINE.GT.NUM.OR.V.EQ.2.) GO TO 40
          WRITE (6,5) L,Y(J),D(L),Z(L),EMP(L),P1(L),P2(L),PL(L),P3(L),P4(L),
          1P6(L),P(L),P5(L)
     5    FORMAT (I11,2F10.2,2F10.3,2F10.3,F10.3,3F10.3,F11.3,F10.3)
          AL=L
          WRITE (7,69)ID1,AL,Y(J),D(L),Z(L),EMP(L),P1(L),P2(L),PL(L),P3(L),
          1P4(L),P6(L),P(L),P5(L)
    20    LINE=LINE+1
          L=L+1
          AL=L
    25    CONTINUE
    26    RETURN
    30    WRITE (6,31) L,Y(J),D(L),Z(L),EMP(L),P1(L),P2(L)
    31    FORMAT (I11,2F10.2,2F10.3,2F10.3)
          WRITE (7,70) ID1,AL,Y(J),D(L),Z(L),EMP(L),P1(L),P2(L)
    70    FORMAT (I5,7F11.3)
          V=1.
          GO TO 20
    40    WRITE (6,41) L,PL(L),P3(L),P4(L),P6(L),P(L),P5(L)
    41    FORMAT (I11,60X,F10.3,3F10.3,F11.3,F10.3)
          WRITE (7,71) ID1,AL,PL(L),P3(L),P4(L),P6(L),P(L),P5(L)
    71    FORMAT (I5,F11.3,66X,6F11.3)
          V=2.
          GO TO 20
    50    ALPHA=0.
          DO  60 J=1,N
          IF (LINE.LT.45) GO TO 59
          LINE=1
          WRITE (6,1) ASTN
          WRITE (6,2)
          WRITE (6,51) ID1,I,M,NX,NNX,XBAR,ALPHA,BETA,GAMMA
          WRITE (7,72) ID1,AI,AJ,ANX,ANNX,XBAR,ALPHA,BETA,GAMMA
    72    FORMAT (I5,8F11.3)
    51    FORMAT (6X,3I7,2I8,4F11.3)
          WRITE (6,4)
    59    WRITE (6,52) J,EMP(J),P1(J),P2(J),PL(J),P3(J),P4(J),P6(J),P(J),
          1P5(J)
          AJ=J
          WRITE (7,73) ID1,AL,EMP(J),P1(J),P2(J),PL(J),P3(J),P4(J),P6(J),
          1P(J),P5(J)
    73    FORMAT (I5,F11.3,33X,9F11.3)
          LINE=LINE+1
          KP(J)=P4(J)*100.
    60    CONTINUE
    52    FORMAT (I11,30X, F10.3,2F10.3,F10.3,3F10.3,F11.3,F10.3)
```

```
FORTRAN IVL27 SOURCE PROGRAM  PRINT     SUBROUTINE             03/25/77    PAGE 0028

          GO TO 26
          END
```

```
FORTRAN IVL27 SOURCE PROGRAM  GAMIT     SUBROUTINE             03/25/77    PAGE 0029

          SUBROUTINE GAMIT (I)
          COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG
          IMPLICIT REAL*8 (A-H,O-Z)
          A=.035868343
          V= -.193527818
          C= .482199394
          D= -.756704078
          E= .918206857
          F= -.897056937
          G=  .988205891
          B= -.577191652
          FLAG=0.
          IF (GAMMA.EQ.1.) GO TO 55
          PEA=GAMMA-1.
          L=PEA+I
          IF (L.LT.1) GO TO 25
          U=PEA+I
          PD=1.
          IF (GAMMA.GT.50) GO TO 45
          IF (L.LT.2) GO TO 16
          LL=L-1
          DO 15 K=1,LL
    15    PD=PD*(U-K)
    16    Y=U-L
          GO TO 30
    25    Y=GAMMA-L
          PD=1./Y
    30    GAM=PD*((((((((A*Y+V)*Y+C)*Y+D)*Y+E)*Y+F)*Y+G)*Y+B)* Y+1.)
    40    RETURN
    55    GAM=1.
          GO TO 40
    45    FLAG=1.
          PD=0.
          DO 50 K=1,L
          PD=PD+ LOG (U-K)
    50    CONTINUE
          Y=U-L
          GAM=((((((((A*Y+V)*Y+C)*Y+D)*Y+E)*Y+F)*Y+G)*Y+B)*Y+1.)
          GAM=PD+LOG(GAM)
          GO TO 40
          END
```

```
      SUBROUTINE PLOT (CHI,INT,G,D,B,P,P6,P4,PL,ASTN,IO1,H,P1,II,JJ,I1,
     1I2,I3,I4,AJJ)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 D,P,PL,P1,P6,P4
      REAL*4 H
      REAL*4 CHI
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PFA,N,IJ,JI,QQ,FLAG,V
      DIMENSION CHI(1),TEM(50),D(1),P(1),P6(1),P4(1),PL(1),H(1),T(50),
     1P1(1)
C I1=1 PLOTS THE DENSITY CURVE
C I2=1 PLOTS THE HISTOGRAM
C I3=1 PLOTS THE CUMULATIVE FREQUENCY
C I4=1 PLOTS THE CUMULATIVE MODEL FIT
      REAL*8 ASTN(8)
      DIMENSION X1(20),Y1(20),X2(20),Y2(20)
      DATA IT/1111/
      W=5.
      AM=2.5
      SF=.6
C INITIALIZE CALCOMP VARIABLES
      IF (IT.NE.1111) GO TO 2
      IT=0
      CALL CALCMP (99,99,24,7)
      CALL CALCMP (99.,0.,3,0)
C CENTER IMAGE OF 9,5X9.5 ONTO A PLOT AREA OF 11X17 INCHES
    2 CALL CALCMP (9.5,9.5,0,13)
      M=0
      LARGE=0
      K=1
      L=0
      DO 8 I=1,NNX
      IF (D(I).EQ.0) GO TO 8
    4 IF (D(I).LE.CHI(K)) GO TO 7
      T(K)=M
      TEM(K)=L
      IF (L.GT.LARGE) LARGE=L
      K=K+1
      L=0
      IF (K-INT) 4,45,45
    7 M=M+1
      L=L+1
    8 CONTINUE
    9 FACT=W/(LARGE+2)
      L=INT-1
      IF (I1.NE.1) GO TO 50
C DRAW BACKGROUND SQUARE
C MOVE WITH BEAM OFF
      CALL CALCMP (AM,AM,0,1)
      CALL CALCMP (W+AM,AM,99,1)
      CALL CALCMP (W+AM,W+AM,99,1)
```



```
      CALL CALCMP (AM,W+AM,99,1)
      CALL CALCMP (AM,AM,99,1)
      DX=AM
      DY=AM
      GP=G-1.
C COMPUTE GAMMA OF GAMMA
      CALL GAMIT(1)
      GG=1./GAM
      XXX=.01
      IF (G.LT.1) FMAX=GG*(EXP(GP*DLOG(XXX)))*EXP(-.01)
      IF (G.GE.1.) FMAX=GG*(EXP(GP*DLOG(GP)))*EXP(-GP)
C ANNOTATE LEFT MARGIN
      D=.25*FMAX/FMAX
      DO 15 I=1,4
      Y=I*O*W+AM
      CALL NUMBER (AM-.4,Y,.07,D*I,0.,2)
      CALL CALCMP (AM-.10,Y,0,1)
      CALL CALCMP (AM,Y,99,1)
      CALL CALCMP (W+AM,Y,0,1)
      CALL CALCMP (W+AM+.1,Y,99,1)
      CALL NUMBER (W+AM+.4,Y,.07,D*I      ,0.,+2)
   15 CONTINUE
      XI=.01
      ZI=CHI(L)/100.
      ZJ=ZI
      DO 25 I=1,100
      Y=(EXP(GP*DLOG(ZI/GP)-ZI+GP))*W+AM
      X=XI+AM
      IF (I.NE.1) GO TO 21
      XI=XI+.04
      ZI=ZI+ZJ
      CALL CALCMP (X,Y,0,1)
      GO TO 25
   21 CALL CALCMP (X,Y,99,1)
      XI=XI+.05
      ZI=ZI+ZJ
   25 CONTINUE
      IF (G.LT.1) FMAX=1000.
C FRAME ADVANCE AFTER WRITING ANNOTATION
      CALL SYMBOL (AM-.35,AM-1.3,.10,ASTN,0.,64)
      CALL SYMBOL (AM+.10,AM-1.5,.07,'  STN   I J      NX    NNX     XBAR
     1  BETA    GAMMA',0.,50)
      CALL NUMBER (AM+.10,AM-1.75,.07,FLOAT(ID1),0.,-1)
      CALL NUMBER (AM+0.6,AM-1.75,.07,FLOAT(II),0.,-1)
      CALL NUMBER (AM+0.8,AM-1.75,.07,FLOAT(JJ),0.,-1)
      CALL NUMBER (AM+1.2,AM-1.75,.07,FLOAT(NX),0.,-1)
      CALL NUMBER (AM+1.6,AM-1.75,.07,FLOAT(NNX),0.,-1)
      CALL NUMBER (AM+2.0,AM-1.75,.07,XBAR,0.,3)
      CALL NUMBER (AM+2.6,AM-1.75,.07,BETA,0.,3)
      CALL NUMBER (AM+3.2,AM-1.75,.07,GAMMA,0.,3)
```

```
      CALL SYMBOL (AM+2.00,AM-1.00,.07,'FMAX',0.,4)
      CALL NUMBER (AM+2.65,AM-1.00,.07,FMAX,0.,3)
      CALL CALCMP (0.,0.,2000,2)
   50 IF (I3.NE.1) GO TO 60
      CALL SYMBOL (AM-.35,AM-1.3,.10,ASTN,0.,64)
      CALL SYMROL (AM+.10,AM-1.5,.07,'  STN   I J      NX    NNX      XBAR
     1  BETA    GAMMA',0.,50)
      CALL NUMBER (AM+.10,AM-1.75,.07,FLOAT(ID1),0.,-1)
      CALL NUMBER (AM+0.6,AM-1.75,.07,FLOAT(II),0.,-1)
      CALL NUMBER (AM+0.8,AM-1.75,.07,FLOAT(JJ),0.,-1)
      CALL NUMBER (AM+1.2,AM-1.75,.07,FLOAT(NX),0.,-1)
      CALL NUMBER (AM+1.6,AM-1.75,.07,FLOAT(NNX),0.,-1)
      CALL NUMBER (AM+2.0,AM-1.75,.07,XBAR,0.,3)
      CALL NUMBER (AM+2.6,AM-1.75,.07,BETA,0.,3)
      CALL NUMBER (AM+3.2,AM-1.75,.07,GAMMA,0.,3)
      CALL SYMBOL (AM-.75,AM+1.90,.07,'FREQUENCY',90.,9)
      CALL SYMBOL (W+AM+.95,AM+1.90,.07,'FREQUENCY',90.,9)
      CALL SYMBOL (AM+2.00,AM-1.00,.07,'PROBABILITY',00.,11)
      CALL CALCMP (AM,AM,0,1)
      CALL CALCMP (W+AM,AM,99,1)
      CALL CALCMP (W+AM,W+AM,99,1)
      CALL CALCMP (AM,W+AM,99,1)
      CALL CALCMP (AM,AM,99,1)
      L=INT-1
      EN=NX
      BNT=EN/INT
      O=W/NX
      YY=D*(EN/INT)
      OX=AM
      DY=Y+AM
      CHIMAX=0.
      DO 55 I=1,L
      IF (CHIMAX.LT.CHI(I)) CHIMAX=CHI(I)
   55 CONTINUE
      DO 10 I=1,L
      X=CHI(I)/CHIMAX*W+AM
      Y=I*YY
      DD=(X-DX)/10.
      XP=DX
      DO 99 J=1,10
      CALL CALCMP (DX,Y+AM,0,1)
      XP=XP+DD
      CALL CALCMP (XP,Y+AM,30,1)
      DX=XP+.03
   99 CONTINUE
      X1(I)=X
      Y1(I)=Y+AM
      FLO=I*BNT
      CALL NUMBER (AM-.4,Y+AM,.07,FLO,0.,1)
      CALL CALCMP (AM-.10,Y+AM,0,1)
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      CALL CALCMP (AM,Y+AM,99,1)
      CALL NUMBER (W+AM+.35,Y+AM,.07,FLO,0.,1)
      CALL CALCMP (W+AM+.1,Y+AM,0,1)
      CALL CALCMP (W+AM,Y+AM,99,1)
   10 CONTINUE
      J=L+6
      JL=L+1
      DO 100 I=JL,J
      Y=Y+YY
      IF (Y.GT.W) GO TO 100
      FLO=FLO+BNT
      CALL NUMBER (AM-.4,Y+AM,.07,FLO,0.,1)
      CALL CALCMP (AM-.10,Y+AM,0,1)
      CALL CALCMP (AM,Y+AM,99,1)
      CALL NUMBER (W+AM+.35,Y+AM,.07,FLO,0.,1)
      CALL CALCMP (W+AM+.1,Y+AM,0,1)
      CALL CALCMP (W+AM,Y+AM,99,1)
  100 CONTINUE
      YP=T(1)*D+AM
      X=AM
      CALL CALCMP (AM,YP,0,1)
      DO 11 I=1,L
      Y=T(I)*D+AM
      CALL CALCMP (X,Y,99,1)
      X=CHI(I)/CHIMAX*W+AM
      CALL CALCMP (X,Y,99,1)
      X2(I)=X
      Y2(I)=Y
      CALL CALCMP (X,AM,0,1)
      CALL CALCMP (X,AM-.10,99,1)
      CALL NUMBER (X-.03,AM-.25,.07,H(I),0.,2)
      CALL CALCMP (X,Y,0,1)
   11 CONTINUE
      AMAX=0.
      MAX=0
      DO 17 I=1,L
      IF (Y1(I).EQ.Y2(I)) GO TO 17
      IF (ABS(Y1(I)-Y2(I)).LT.AMAX) GO TO 16
      AMAX=ABS(Y1(I)-Y2(I))
      MAX=I
   16 DX=AM
      IF (I.NE.1) DX=X1(I-1)
      IF (Y1(I).GT.Y2(I)) GO TO 14
      D=(Y2(I)-Y1(I))/6
      DY=Y1(I)+D
      DO 12 J=1,5
      CALL CALCMP (DX,DY,0,1)
      CALL CALCMP (X1(I),DY,99,1)
      DY=DY+D
   12 CONTINUE
```

```
 17     CONTINUE
        GO TO 22
 14     D=(XI(I)-DX)/6
        DX=DX+D
        DO 18 J=1,5
        CALL CALCMP (DX,Y1(I),0,1)
        CALL CALCMP (DX,Y2(I),99,1)
        DX=DX+D
 18     CONTINUE
        GO TO 17
 22     DX=AM
        IF (MAX.NE.1) DX=X1(MAX-1)
        IF (Y1(MAX).EQ.Y2(MAX)) GO TO 13
        IF (Y2(MAX).GT.Y1(MAX)) GO TO 20
        D=(Y1(MAX)-Y2(MAX))/6
        DY=Y2(MAX)+D
        DO 19 J=1,5
        CALL CALCMP (DX,DY,0,1)
        CALL CALCMP (X1(MAX),DY,99,1)
        DY=DY+D
 19     CONTINUE
 20     D=(X1(MAX)-DX)/6
        DX=DX+D
        DO 23 J=1,5
        CALL CALCMP (DX,Y1(MAX),0,1)
        CALL CALCMP (DX,Y2(MAX),99,1)
        DX=DX+D
 23     CONTINUE
 13     CALL CALCMP (0.,0.,2000,2)
 60     IF (I2.NE.1) GO TO 70
C MAKE A FRAME WITH HISTOGRAM ONLY
        EN=NX
        CALL CALCMP (AM,AM,0,1)
        CALL CALCMP (W+AM,AM,99,1)
        CALL CALCMP (W+AM,W+AM,99,1)
        CALL CALCMP (AM,W+AM,99,1)
        CALL CALCMP (AM,AM,99,1)
        CALL SYMBOL (AM-.35,AM-1.3,.10,ASTN,0.,64)
        CALL SYMBOL (AM+.10,AM-1.5,.07,'  STN   I J      NX    NNX      XBAR
       1  BETA    GAMMA',0.,50)
        CALL NUMBER (AM+.10,AM-1.75,.07,FLOAT(IOI),0.,-1)
        CALL NUMBER (AM+0.6,AM-1.75,.07,FLOAT(II),0.,-1)
        CALL NUMBER (AM+0.8,AM-1.75,.07,FLOAT(JJ),0.,-1)
        CALL NUMBER (AM+1.2,AM-1.75,.07,FLOAT(NX),0.,-1)
        CALL NUMBER (AM+1.6,AM-1.75,.07,FLOAT(NNX),0.,-1)
        CALL NUMBER (AM+2.0,AM-1.75,.07,XBAR,0.,3)
        CALL NUMBER (AM+2.6,AM-1.75,.07,BETA,0.,3)
        CALL NUMBER (AM+3.2,AM-1.75,.07,GAMMA,0.,3)
        CALL SYMBOL (AM-.75,AM+1.90,.07,'FREQUENCY',90.,9)
        CALL SYMBOL (W+AM+.75,AM+1.90,.07,'FREQUENCY',90.,9)
```



```
        CALL SYMBOL (AM+2.00,AM-1.00,.07,'PROBABILITY',00.,11)
C DRAW THE THEORETICAL LINE
        Y=FACT*(EN/INT)+AM
        XI=.05+AM
        DO 26 I=1,49
        CALL CALCMP (XI,Y, 0,1)
        CALL CALCMP (XI+.03,Y,99,1)
 26     XI=XI+.10
        DX=AM
        DO 27 I=1,L
        X=CHI(I)/CHIMAX*W+AM
        Y=TEM(I)*FACT+AM
        CALL CALCMP (DX,AM,0,1)
        CALL CALCMP (DX,Y,99,1)
        CALL CALCMP (X,Y,99,1)
        CALL CALCMP (X,AM,99,1)
        DX=X
        CALL CALCMP (X,AM,0,1)
        CALL CALCMP (X,AM-.10,99,1)
        CALL NUMBER (X-.03,AM-.25,.07,H(I),0.,2)
 27     CONTINUE
        J=NX/INT+5
        DO 29 I=1,J
        Y=FACT*I+AM
        IF (Y.GT.(W+AM)) GO TO 29
        CALL NUMBER (AM-.4,Y,.07,FLOAT(I),0.,-1)
        CALL CALCMP (AM-.10,Y,0,1)
        CALL CALCMP (AM,Y,99,1)
        CALL CALCMP (W+AM,Y,0,1)
        CALL CALCMP (W+AM+.1,Y,99,1)
        CALL NUMBER (W+AM+.4,Y,.07,FLOAT(I),0.,-1)
 29     CONTINUE
        CALL CALCMP (0.,0.,2000,2)
 70     IF (I4.NE.1) GO TO 80
C DRAW BACKGROUND SQUARE OF 5X5 INCHES
        CALL CALCMP (AM,AM,0,1)
        CALL CALCMP (W+AM,AM,99,1)
        CALL CALCMP (W+AM,W+AM,99,1)
        CALL CALCMP (AM,W+AM,99,1)
        CALL CALCMP (AM,AM,99,1)
C DRAW 45 DEGREE LINE
        CALL CALCMP (AM,AM,0,1)
        CALL CALCMP (W+AM,W+AM,28,1)
        KK=NNX
        DO 261I=1,NNX
        PCP=D(KK)
        IF (D(KK).LT.AJJ) GO TO 271
        KK=KK-1
 261    CONTINUE
 271    PCP=PCP/B
```

```
      IF (PCP.LT.P1(KK)) PCP=P1(KK)
      INC=PCP+1.5
      Y=W/INC
      X=Y
      DO 30 I=1,NNX
      IF (D(I).LE.0.) GO TO 30
      IF (D(I).GE.AJJ) GO TO 30
      YP=(D(I)/8*Y)+AM
      XP=P1(I)*X+AM
C PLOT THE DATA/BETA VALUES TAKEN FROM COL 4
      CALL SYMBOL (XP,YP,.03, 4,0.,-1)
   30 CONTINUE
C ANNOTATE THE LEFT MARGIN AND THE RIGHT MARGIN
      YI=Y
      IS=(PCP+1.5)/10.
      IS2=IS*.5
      YIC=YI*IS*.5
      Y=YIC+AM
      JS=IS+25
      DO 33 I=1,JS
      IF (Y.GT.(W+AM)) GO TO 34
      CALL CALCMP (AM,Y,0,1)
      CALL CALCMP (AM+.1,Y,99,1)
      Y=Y+YIC
      IF (Y.GT.(W+AM)) GO TO 34
      CALL CALCMP (AM,Y,0,1)
      CALL CALCMP (AM-.1,Y,99,1)
      A=I*IS
      CALL NUMBER (AM-.4,Y,.10,A,0.,+1)
      Y=Y+YIC
   33 CONTINUE
   34 YI=INC*B
      INC=YI
      AS=YI/10.
      YIC=(W/YI)*AS*.5
      Z=YIC+AM
      JS=AS+25
      DO 35 I=1,JS
      IF (Z.GT.(W+AM)) GO TO 36
      CALL CALCMP (AM+W,Z,0,1)
      CALL CALCMP (AM+W-.1,Z,99,1)
      Z=Z+YIC
      IF (Z.GT.(W+AM)) GO TO 36
      CALL CALCMP (AM+W,Z,0,1)
      CALL CALCMP (AM+W+.1,Z,99,1)
      A=I*AS
      CALL NUMBER (AM+W+.2,Z,.10,A,0.,+1)
      Z=Z+YIC
   35 CONTINUE
   36 U=W+AM
```



```
      CALL SYMBOL (AM-.75,AM+1.9,.07,'QUANTILES',90.,9)
      CALL SYMBOL (U+.75,AM+1.9,.07,'QUANTITIES',90.,10)
C ANNOTATE BOTTOM SCALES
      XI=X
      KK=2
      DO 38 I=1,N
      X=(P(I)/B)*XI+AM
      IF (X.GE.(W+AM)) GO TO 40
C INSIDE SCALE DETERMINED BY COL 11 AGAINST COL 12/BETA
      CALL CALCMP (X,AM,0,1)
      CALL CALCMP (X,AM+.03,99,1)
      IF (P6(I).LT..5) GO TO 38
      CALL NUMBER (X-.03,AM+.10,.05,P6(I),0.,3)
   38 CONTINUE
   40 KK=0
      DO 44 I=1,N,2
      X=(P4(I)/8)*XI+AM
      IF (X.GE.(W+AM)) GO TO 46
      CALL CALCMP (X,AM,0,1)
      IF (KK.EQ.1) GO TO 37
      KK=1
      CALL CALCMP (X,AM-.14,99,1)
      CALL NUMBER (X-.03,AM-.25,.05,PL(I),0.,3)
      GO TO 44
   37 CALL CALCMP (X,AM-.14,99,1)
      CALL NUMBER (X-.03,AM-.45,.05,PL(I),0.,3)
      KK=0
   44 CONTINUE
   46 CALL SYMBOL (AM+2.00,AM-1.00,.07,' PROBABILITY ',0.,13)
      CALL SYMBOL (AM-.35,AM-1.3,.10,ASTN,0.,64)
      CALL SYMBOL (AM+.10,AM-1.5,.07,'  STN   I J     NX    NNX     XBAR
     1  BETA    GAMMA',0.,50)
      CALL NUMBER (AM+.10,AM-1.75,.07,FLOAT(IO1),0.,-1)
      CALL NUMBER (AM+0.6,AM-1.75,.07,FLOAT(II),0.,-1)
      CALL NUMBER (AM+0.8,AM-1.75,.07,FLOAT(JJ),0.,-1)
      CALL NUMBER (AM+1.2,AM-1.75,.07,FLOAT(NX),0.,-1)
      CALL NUMBER (AM+1.6,AM-1.75,.07,FLOAT(NNX),0.,-1)
      CALL NUMBER (AM+2.0,AM-1.75,.07,XBAR,0.,3)
      CALL NUMBER (AM+2.6,AM-1.75,.07,BETA,0.,3)
      CALL NUMBER (AM+3.2,AM-1.75,.07,GAMMA,0.,3)
      CALL CALCMP (0.,0.,2000,2)
   80 RETURN
   45 I=K-1
      TEM(K)=NX-T(I)
      GO TO 9
      END
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(Continued from inside front cover)

EDS 16 NGSDC 1 - Data Description and Quality Assessment of Ionospheric Electron Density Profiles for ARPA Modeling Project. Raymond O. Conkright, in press, 1976.

EDS 17 GATE Convection Subprogram Data Center: Analysis of Ship Surface Meteorological Data Obtained During GATE Intercomparison Periods. Fredric A. Godshall, Ward R. Seguin, and Paul-Sabol, October 1976. (PB-263-000)

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TECHNICAL REPORTS—Journal quality with extensive details, mathematical developments, or data listings.

TECHNICAL MEMORANDUMS — Reports of preliminary, partial, or negative research or technology results, interim instructions, and the like.

CONTRACT AND GRANT REPORTS—Reports prepared by contractors or grantees under NOAA sponsorship.

TECHNICAL SERVICE PUBLICATIONS—These are publications containing data, observations, instructions, etc. A partial listing: Data serials; Prediction and outlook periodicals; Technical manuals, training papers, planning reports, and information serials; and Miscellaneous technical publications.

ATLAS—Analysed data generally presented in the form of maps showing distribution of rainfall, chemical and physical conditions of oceans and atmosphere, distribution of fishes and marine mammals, ionospheric conditions, etc.

Information on availability of NOAA publications can be obtained from:

ENVIRONMENTAL SCIENCE INFORMATION CENTER
ENVIRONMENTAL DATA SERVICE
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3300 Whitehaven Street, N.W.
Washington, D.C. 20235

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